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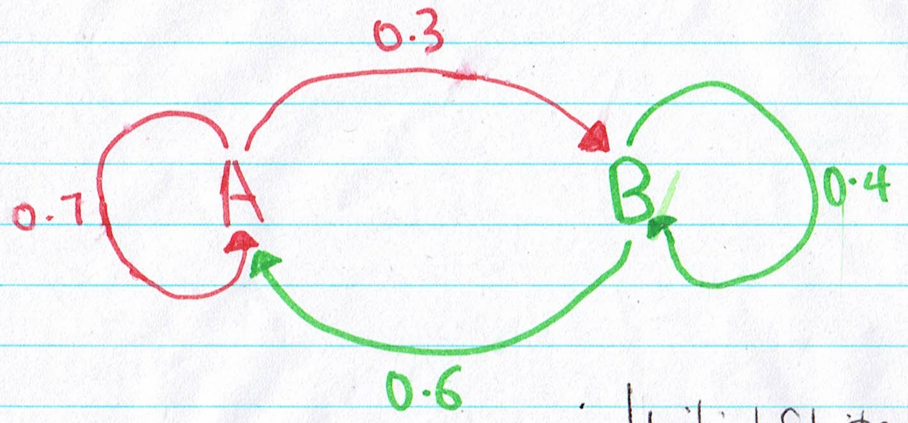
Markov Matrices



$$M = \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix} \end{array}$$

1st 3rd

$$M^2 \begin{bmatrix} 0.67 & 0.66 \\ 0.33 & 0.34 \end{bmatrix}$$

2nd 4th

$$M^3 \begin{bmatrix} 0.667 & 0.666 \\ 0.333 & 0.334 \end{bmatrix}$$

$$M^4 = \begin{bmatrix} 0.6667 & 0.6666 \\ 0.3333 & 0.3334 \end{bmatrix}$$

$$M^5 = \begin{bmatrix} 0.66667 & 0.66666 \\ 0.33333 & 0.33334 \end{bmatrix}$$

$$M^{10} = \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.666667 & 0.666667 \\ 0.333333 & 0.333333 \end{bmatrix} \end{array}$$

Initial State

Monday: Ate An Apple

$$V_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\text{Monday} \xrightarrow{M} \text{Tuesday} \xrightarrow{M} \text{Wed}$$

State: V_0 (initial) $\quad V_1 \quad V_2$

$V_1 = M \times V_0$

$V_2 = M \times V_1$

$= (M) \times (M \times V_0)$

$= M^2 \times V_0$

$$\begin{bmatrix} 0.67 & 0.66 \\ 0.33 & 0.34 \end{bmatrix} \times \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$V_2 = \begin{pmatrix} 0.67 \\ 0.33 \end{pmatrix} \begin{array}{c} A \\ B \end{array}$$

P(on wed eat an apple | given that mon ate an apple) = 0.67

P(on wed eat an banana | given that mon ate an apple) = 0.33

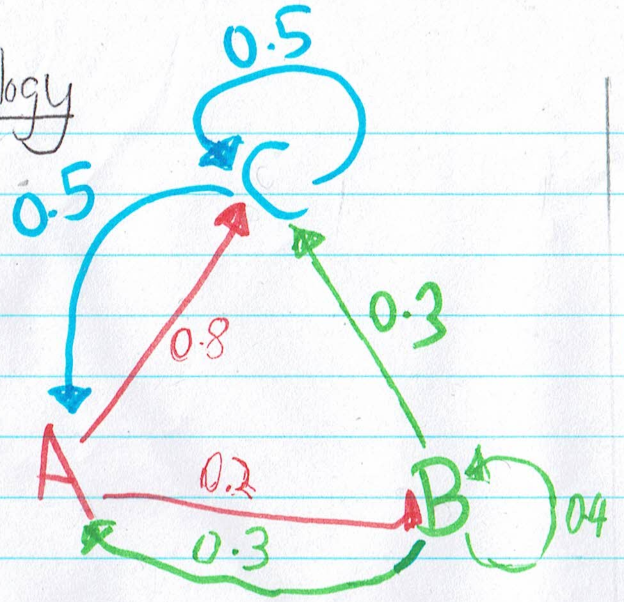
From Excel graphs; as power increase → ∞

1st element approach value 0.66666 7

2nd 0.333333

3rd 0.666667

4th 0.333333



Transition Matrix

$$M = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 0 & 0.3 & 0.5 \\ 0.2 & 0.4 & 0 \\ 0.8 & 0.3 & 0.5 \end{pmatrix} \end{array}$$

Initial State: Monday: Ate an apple

$$V_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\underbrace{\text{Mon} \xrightarrow{M} \text{Tue} \xrightarrow{M} \text{Wed} \xrightarrow{M} \text{Thu} \xrightarrow{M}}_{M^4} \text{Fri} \rightarrow V_4$$

$$V_4 = \underline{M^4} \times V_0$$

$$= \begin{array}{c} A \\ B \\ C \end{array} \begin{pmatrix} 0 \\ 0.2 \\ 0.8 \end{pmatrix}$$

Meaning that on friday, the probability (p) is p(on friday eat an apple | given that Monday ate an apple) = 0

p(on Friday eat an banana | given that Monday ate an apple) = 0.2

p (on Friday eat a cake/clemetine | given that Monday ate an apple) = 0.8

$$M^{\infty} = \begin{array}{c} A \\ B \\ C \end{array} \begin{pmatrix} 0.319149 & 0.319149 & 0.319149 \\ 0.106383 & 0.106383 & 0.106383 \\ 0.574468 & 0.574468 & 0.574468 \end{pmatrix} \Rightarrow$$

On average, the probability of eating:

An Apple is 0.319149

A banana is 0.106383

A cake is 0.574468

This means, on average, the probability of eating an apple every day is 0.666667 and the probability of eating an banana is 0.333333