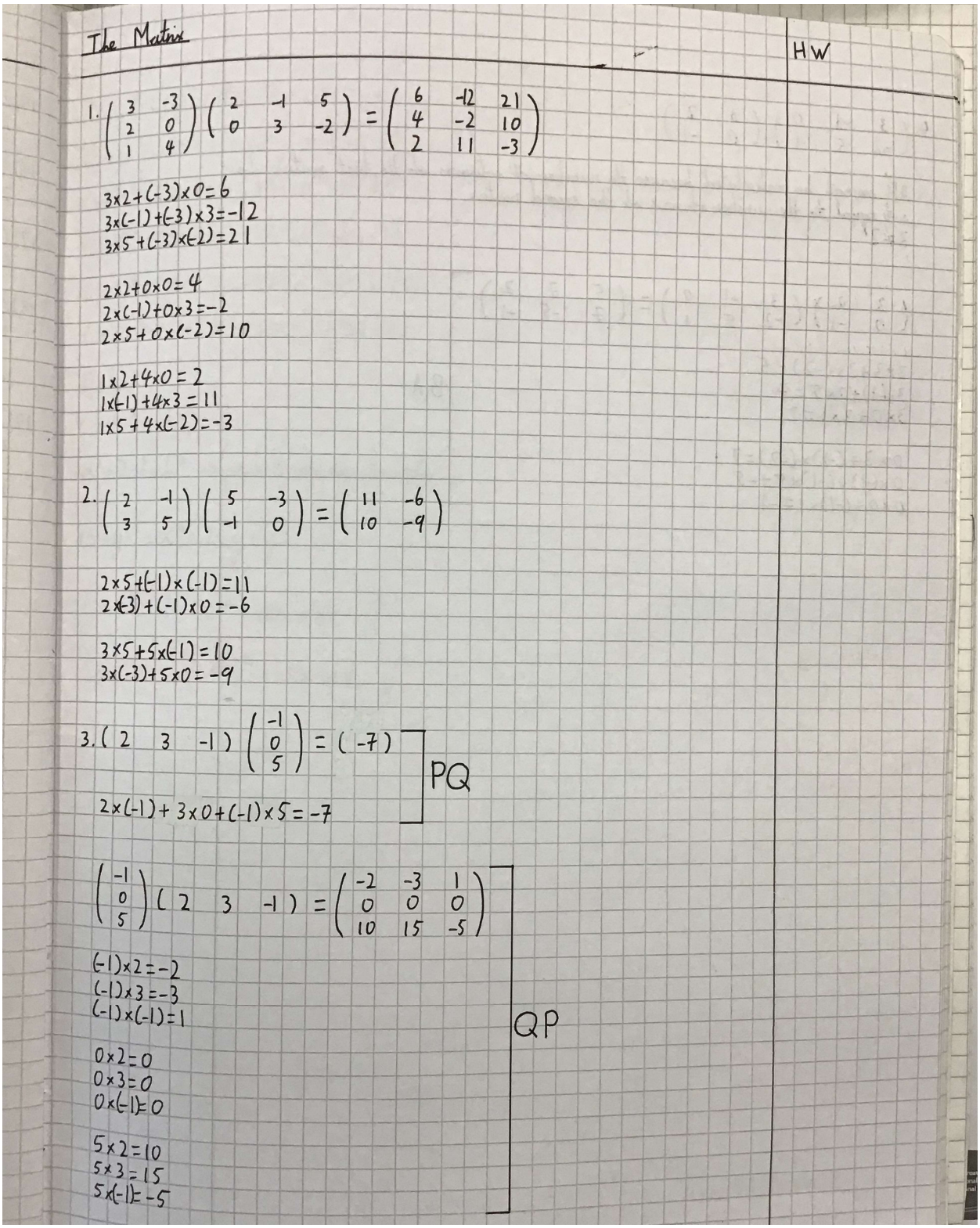

The Matrix

HW

1. $\begin{pmatrix} 3 & -3 \\ 2 & 0 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 & 5 \\ 0 & 3 & -2 \end{pmatrix} = \begin{pmatrix} 6 & -12 & 21 \\ 4 & -2 & 10 \\ 2 & 11 & -3 \end{pmatrix}$

$3\times2+(-3)\times0=6$
$3\times(-1)+(-3)\times3=-12$
$3\times5+(-3)\times(-2)=21$

$2\times2+0\times0=4$
$2\times(-1)+0\times3=-2$
$2\times5+0\times(-2)=10$

$1\times2+4\times0=2$
$1\times(-1)+4\times3=11$
$1\times5+4\times(-2)=-3$

2. $\begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} 5 & -3 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 11 & -6 \\ 10 & -9 \end{pmatrix}$

$2\times5+(-1)\times(-1)=11$
$2\times(-3)+(-1)\times0=-6$

$3\times5+5\times(-1)=10$
$3\times(-3)+5\times0=-9$

3. $\begin{pmatrix} 2 & 3 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} -7 \end{pmatrix}$ PQ

$2\times(-1)+3\times0+(-1)\times5=-7$

$\begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}\begin{pmatrix} 2 & 3 & -1 \end{pmatrix} = \begin{pmatrix} -2 & -3 & 1 \\ 0 & 0 & 0 \\ 10 & 15 & -5 \end{pmatrix}$ QP

$(-1)\times2=-2$
$(-1)\times3=-3$
$(-1)\times(-1)=1$

$0\times2=0$
$0\times3=0$
$0\times(-1)=0$

$5\times2=10$
$5\times3=15$
$5\times(-1)=-5$

4. $\begin{pmatrix} 3 & -1 & 0 \\ -2 & 5 & 1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix}$

AB cannot be calculated because the number of columns of the first matrix does not equal to the number of rows of the second matrix.
$3 \neq 2$

$\begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 & -1 & 0 \\ -2 & 5 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 7 & 2 \\ 2 & -5 & -1 \end{pmatrix}$

$3\times3+2\times(-2)=5$
$3\times(-1)+2\times5=7$
$3\times0+2\times1=2$

$0\times3+(-1)\times(-2)=2$
$0\times(-1)+(-1)\times5=-5$
$0\times0+(-1)\times1=-1$

BA

The Matrix

HW

1. $A=\begin{pmatrix}1&2\\3&4\end{pmatrix}$ $B=\begin{pmatrix}4&3\\2&1\end{pmatrix}$

$AB=\begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}4&3\\2&1\end{pmatrix}=\begin{pmatrix}8&5\\20&13\end{pmatrix}$

$BA=\begin{pmatrix}4&3\\2&1\end{pmatrix}\begin{pmatrix}1&2\\3&4\end{pmatrix}=\begin{pmatrix}13&20\\5&8\end{pmatrix}$

$AB\neq BA$

$A=\begin{pmatrix}0&0\\0&0\end{pmatrix}$ $B=\begin{pmatrix}1&2\\3&4\end{pmatrix}$

$AB=\begin{pmatrix}0&0\\0&0\end{pmatrix}\begin{pmatrix}1&2\\3&4\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

$BA=\begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}0&0\\0&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

$AB=BA$

In some cases $AB=BA$ but not in all cases.

2. $A=\begin{pmatrix}1&2\\3&4\end{pmatrix}$ $B=\begin{pmatrix}-4&-3\\-2&-1\end{pmatrix}$

$AB=\begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}-4&-3\\-2&-1\end{pmatrix}=\begin{pmatrix}-8&-5\\-20&-13\end{pmatrix}$

$AB\neq 0$

$A=\begin{pmatrix}1&2\\3&4\end{pmatrix}$ $B=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

$AB=\begin{pmatrix}1&2\\3&4\end{pmatrix}\begin{pmatrix}0&0\\0&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

$AB=0$

If $AB=0$ at least one of A or B must be must equal to 0.

3. $M=\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ $M^2=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$ $M^3=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ $M^4=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

$M^5=\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ $M^6=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$ $M^7=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ $M^8=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

Every 4 is a cycle.

$2023\div 4=505\text{ r }3$

$M^{2023}=M^3$

$$\boxed{M^{2023}=\begin{pmatrix}0&1\\-1&0\end{pmatrix}}$$