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if $\theta + \phi + \psi = \frac{\pi}{2}$

$\sin(\frac{\pi}{2}) = 1$

$\sin(\theta + \phi + \psi) = 1$

$$\boxed{\begin{aligned}&\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A \\ &\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B\end{aligned}}$$

① $\therefore \sin[(\theta + \phi) + (\psi)] = \sin(\theta + \phi)\cos\psi + \sin\psi\cos(\theta + \phi)$

$\theta + \phi + \psi = \frac{\pi}{2} \Rightarrow \theta + \phi = \frac{\pi}{2} - \psi$

$\therefore \sin(\theta + \phi) = \sin(\frac{\pi}{2} - \psi)$

may be seen because cos could be thought of as a transformation of the sine graph by $\frac{\pi}{2}$ shift in x However it can be confusing →

$\sin(\frac{\pi}{2} - \psi) = \sin\frac{\pi}{2}\cos\psi - \sin\psi\cos\frac{\pi}{2}$

$= 1(\cos\psi) - 0 = \underline{\cos\psi}$

① $\sin(\theta + \phi + \psi) = \cos^2\psi + \sin\psi\cos(\theta + \phi)$

$\boxed{\sin^2\psi + \cos^2\psi = 1 \Rightarrow \cos^2\psi = 1 - \sin^2\psi}$

$\boxed{\text{As shown previously}}$ → $1 = 1 - \sin^2\psi + \sin\psi\cos(\theta + \phi)$

$0 = -\sin^2\psi + \sin\psi\cos(\theta + \phi)$

$0 = \sin\psi(-\sin\psi + \cos(\theta + \phi))$

if $\sin\psi = 0 \Rightarrow \psi = 0$ for $0 \le \psi \le \frac{\pi}{2}$

Assume $\psi \neq 0$ $(\theta, \phi, \psi > 0)$

$\therefore$ we can omit solution

hence:

$0 = -\sin\psi + \cos(\theta + \phi)$

use angle sum identity for $\cos(A \pm B)$

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$0 = -\sin\psi + \underbrace{\cos\theta\cos\phi - \sin\theta\sin\phi}$$

Rearrange

$$\sin\psi + \sin\theta\sin\phi = \cos\theta\cos\phi$$

$$(\sin\psi + \sin\theta\sin\phi)^2 = \cos^2\theta\cos^2\phi$$

$$\sin^2\theta + \cos^2\theta \equiv 1 \Rightarrow \cos^2\theta \equiv 1 - \sin^2\theta$$

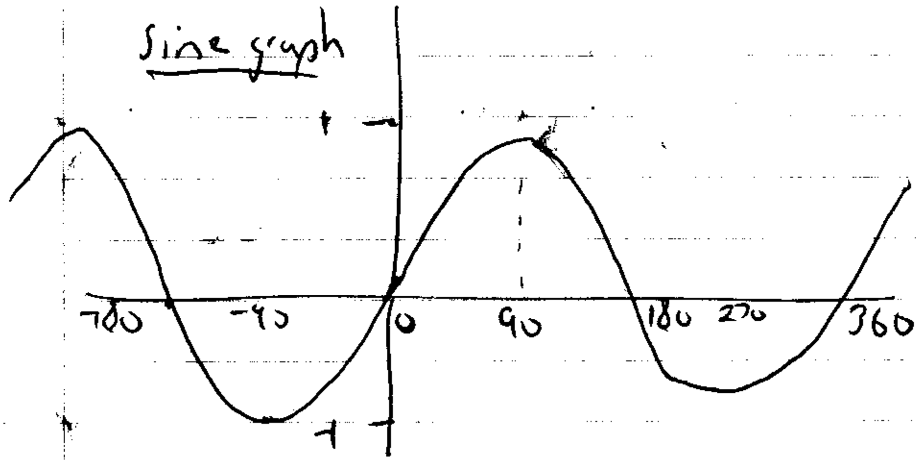
$$\sin^2\psi + 2\sin\theta\sin\phi\sin\psi + \sin^2\theta\sin^2\phi = (1-\sin^2\theta)(1-\sin^2\phi)$$

$$\sin^2\psi + 2\sin\theta\sin\phi\sin\psi + \cancel{\sin^2\theta\sin^2\phi} = 1 - \sin^2\theta - \sin^2\phi + \cancel{\sin^2\theta\sin^2\phi}$$

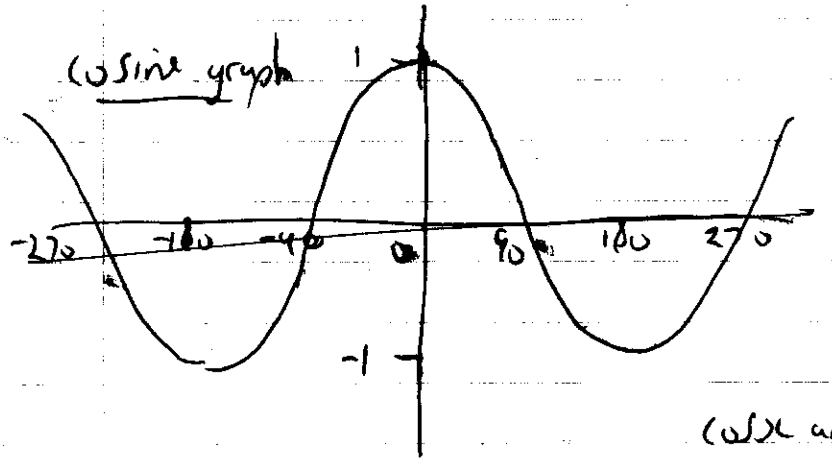
$$\sin^2\theta + \sin^2\phi + \sin^2\psi + 2\sin\theta\sin\phi\sin\psi = 1$$

Alternative method to show $\sin(\frac{\pi}{2} - \psi) = \cos\psi$

Sine graph



cosine graph



I've sketched in degrees but you can see cos graph to be the sine graph but shifted by 90 to the left

$\cos x$ and $\sin x$ are just functions of x

$\therefore f(x+90)$ shifts to the left

$$\Rightarrow \cos(x) = \sin(x+90)$$

if $x = -x$ $\quad \cos(x) = \cos(-x)$ $\because$ of the symmetry of the cosine graph

$$\therefore \cos(-x) = \sin(-x+90)$$
$$\cos(x) = \sin(90-x)$$
$$\cos(\psi) = \sin(\frac{\pi}{2} - \psi)$$ now in radians

6. $\theta = \phi = \frac{1}{5}\pi$

$\Rightarrow \frac{1}{5}\pi + \frac{1}{5}\pi + \psi = \frac{\pi}{2}$

$\psi = \frac{\pi}{10}$

$\boxed{\frac{\pi}{10} \times 2 = \frac{1}{5}\pi}$

$$\boxed{\begin{aligned} \therefore \quad \theta &= 2\psi \\ \phi &= 2\psi \\ \psi &= \psi \end{aligned}}$$

Solving in terms of ψ

As shown earlier: $\sin^2\theta + \sin^2\phi + \sin^2\psi + 2\sin\theta\sin\phi\sin\psi - 1 = 0$

$\Rightarrow \sin^2(2\psi) + \sin^2(2\psi) + \sin^2\psi + 2\sin(2\psi)\sin(2\psi)\sin\psi - 1 = 0$

$2\sin^2(2\psi) + \sin^2\psi + 2\sin^2(2\psi)\sin\psi - 1 = 0$

$\sin^2(2\psi) = (\sin(2\psi))^2$ can use angle sum identity for sine

$$\left[\sin(\psi+\psi)\right]^2 = \left[\sin\psi\cos\psi + \sin\psi\cos\psi\right]^2$$

$$= (2\sin\psi\cos\psi)^2$$

$$= 4\sin^2\psi\cos^2\psi$$

use $\sin^2\theta + \cos^2\theta \equiv 1$

$$= 4\sin^2\psi(1-\sin^2\psi)$$

$$= 4\sin^2\psi - 4\sin^4\psi$$

$$2\left[4\sin^2\psi - 4\sin^4\psi\right] + \sin^2\psi + 2\sin\psi\left[4\sin^2\psi - 4\sin^4\psi\right] - 1 = 0$$

$$\underline{8\sin^2\psi} - 8\sin^4\psi + \underline{\sin^2\psi} + 8\sin^3\psi - 8\sin^5\psi - 1 = 0$$

$$\underline{\underline{\text{let } x = \sin\psi}}$$

$$-8x^5 - 8x^4 + 8x^3 + 9x^2 - 1 = 0$$

Attempt polynomial division. I will try $(x-1)$ as a factor – because there is a -1 in the equation which must be produce by multiplication with 1's and -1's. The coefficient of x is 1 because in the question the x^3 coefficient is 8 and I want to keep that in my division $\frac{8}{1} = 8$

hence:

$$
\begin{array}{r}
-8x^4 - 16x^3 - 8x^2 + x + 1 \\
(x-1)\;\overline{)\,-8x^5 - 8x^4 + 8x^3 + 9x^2 + 0x - 1} \\
-\;\underline{-8x^5 + 8x^4} \\
-16x^4 + 8x^3 \\
-\;\underline{-16x^4 + 16x^3} \\
-8x^3 + 9x^2 \\
-\;\underline{-8x^3 + 8x^2} \\
x^2 + 0x \\
-\;\underline{x^2 - x} \\
x - 1 \\
-\;\underline{x - 1} \\
0
\end{array}
$$

$$(x-1)(-8x^4 - 16x^3 - 8x^2 + x + 1)$$

The highest power is 4. highest power is 3. in $8x^3+8x^2-1=0$ $\therefore$ we must perform one more division

continue with x coefficient as 1 for same reason
using factor theorem on $f(x) = -8x^4-16x^3-8x^2+x+1$
For $x=1$ and $x=-1$ (1 and -1 multiply to make 1 in some combination)
$f(-1)=0 \Rightarrow (x+1)$ is a factor

$$
\begin{array}{r l}
 & -8x^3 - 8x^2 + 0x + 1 \\
\hline
(x+1) \,\big) & -8x^4 - 16x^3 - 8x^2 + x + 1 \\
- & -8x^4 - 8x^3 \\
\hline
 & -8x^3 - 8x^2 \\
- & -8x^3 - 8x^2 \\
\hline
 & 0 + x \\
- & 0 + 0 \\
\hline
 & x + 1 \\
- & x + 1 \\
\hline
 & 0
\end{array}
$$

$\Rightarrow (x-1)(x+1)(-8x^3-8x^2+1) = 0$
one of these brackets must equal 0 for LHS $= 0$

$\therefore$ one out of solutions is when $-8x^3-8x^2+1=0$

$$\boxed{8x^3+8x^2-1=0}$$

The equation has been formed from an equation we know to be True. When $x = \sin\psi$ $\boxed{\psi = \frac{1}{10}\pi}$
$x = \sin(\frac{1}{10}\pi) \Rightarrow \sin\frac{1}{10}\pi$ satisfies the equation $8x^3+8x^2-1=0$