

The Koch's snowflake

Table 1

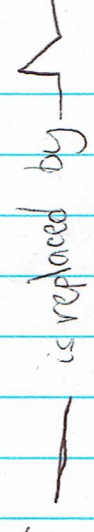
Iteration	No. of sides	No. of new triangles added	Notes
1	3		
2	3×4 $\rightarrow 3 \times 4 \times 4$ $= 3 \times 4^2$	$3 \times 4 \times 1$ $\leftarrow (2-2)$	each side is replaced by  becomes 4 new sides add 1 new triangle
3	$3 \times 4 \times 4$ $= 3 \times 4^3$	$3 \times 4 \times 4 \times 1$ $\leftarrow (3-2)$	
4	$3 \times 4 \times 4 \times 4$ $= 3 \times 4^4$	$3 \times 4 \times 4 \times 4 \times 1$ $= 3 \times 4^3$ $\leftarrow (4-2)$	
n	$3 \times 4^{n-1}$	$3 \times 4^{n-2}$	

Table 2 About sizes: Length and area, Ratio notes

Iteration	No. of Sides	Length of side	Perimeter No. of sides \times Length of side	Area of triangle	Nb. of triangle added	Area of each new triangle Δ	Area added to the Koch Snowflake	Total Koch Snowflake area
1	3	L	3L	A	0	0	0	A
2	3×4	$\frac{1}{3}L$	$3 \times 4 \times \frac{1}{3}L$	$\frac{1}{9}A$	3x	$\frac{1}{9}A$	$3 \times 1 \times \frac{1}{9}A$	$A + 3 \times 1 \times \frac{1}{9}A$
3	3×4^2	$\frac{1}{3} \times \frac{1}{3} \times L$ $(\frac{1}{3})^2 \times L$	$3 \times 4^2 \times (\frac{1}{3})^2 \times L$	$(\frac{1}{9})^2 A$	$3 \times 4 \times 1$	$\frac{1}{9} \times \frac{1}{9} A$	$3 \times 4 \times 1 \times (\frac{1}{9})^2 A$	$1 + 3 \times 1 \times \frac{1}{9} A + 3 \times 4 \times 1 \times (\frac{1}{9})^2 A$
4	3×4^3	$(\frac{1}{3})^3 \times L$	$3 \times 4^3 \times (\frac{1}{3})^3 \times L$	$(\frac{1}{9})^3 A$	$3 \times 4^2 \times 1$	$(\frac{1}{9})^3 A$	$3 \times 4^2 \times 1 \times (\frac{1}{9})^3 A$	$A + 3 \times 1 \times \frac{1}{9} A + 3 \times 4 \times 1 \times (\frac{1}{9})^2 A + 3 \times 4^2 \times 1 \times (\frac{1}{9})^3 A$
n	$3 \times 4^{n-1}$	$(\frac{1}{3})^{n-1} \times L$	$3 \times 4^{n-1} \times (\frac{1}{3})^{n-1} \times L$	$(\frac{1}{9})^{n-1} A$	$3 \times 4^{n-2} \times 1$	$(\frac{1}{9})^{n-1} A$	$3 \times 4^{n-2} \times 1 \times (\frac{1}{9})^{n-1} A$	$A + 3 \times 1 \times \frac{1}{9} A + 3 \times 4 \times 1 \times (\frac{1}{9})^2 A + 3 \times 4^2 \times 1 \times (\frac{1}{9})^3 A + \dots + 3 \times 4^{n-2} \times 1 \times (\frac{1}{9})^{n-1} A$

Conclusions:

- Perimeter of Koch Snowflake at the n^{th} iteration is $3 \times 4^{n-1} \times (\frac{1}{3})^{n-1} \times L$
- Area of Koch Snowflake at n^{th} iteration is:

$$\begin{aligned}
 & A + 3 \times 4 \times 1 \times (\frac{1}{9}) A + 3 \times 4 \times 1 \times (\frac{1}{9})^2 A + 3 \times 4^2 \times 1 \times (\frac{1}{9})^3 A + \dots + 3 \times 4^{n-2} \times 1 \times (\frac{1}{9})^{n-1} A \\
 & = (1 + 3 \times 4 \times 1 \times (\frac{1}{9}) + 3 \times 4 \times 1 \times (\frac{1}{9})^2 + 3 \times 4^2 \times 1 \times (\frac{1}{9})^3 + \dots + 3 \times 4^{n-2} \times 1 \times (\frac{1}{9})^{n-1}) \times A
 \end{aligned}$$

I used excel to calculate the equation above *Multiplying factor*

NOTES

$\triangle ABC$ and $\triangle DEF$
 are the same

The small \triangle is $\frac{1}{9}$ of the big \triangle
 The side of the small \triangle is $\frac{1}{3}$ of original \triangle

