

a) There are 6 elves in total, E1, E2, E3, B1, B2, B3.

If the elves are in a line (not a circle), isolating 1 elf from the list means that for example, E1 has an equal chance of sitting with either E2, E3, B1, B2, or B3. Allocating the best friend of the elf according to their number means that E1's best friend is B1. If this is the case this means that there is a $\frac{1}{5}$ chance that E1 sits with B1.

If E1 is then sitting with B1, this means that E2, E3, B2, B3 are left.

There is a $\frac{1}{3}$ chance that E2 will sit next to B2.

And now there is a $\frac{1}{1}$ chance that E3 will sit next to B3.

Multiplying all the probabilities together will get $\frac{1}{15}$ that all the elves will sit next to their besties in a straight line. However, since they are sitting on a round table, the probability would double since everyone would now be sitting next to 2 others, therefore $\frac{2}{15}$ chance of sitting next to their bestie.

b)

For 2 pairs of an elf and their bestie to be sitting next to each other while a pair of an elf and their bestie to be separated there is only 1 configuration. There must be a 2-seat gap between the elf and their bestie therefore, an elf and their bestie must be sitting opposite each other.

Considering that the 2 pairs would be arranged in either way of the elf and the arrangement of seating of those remaining pairs could be in 4 different ways (E2B2|E3B3, E2B2|B3E3, B2E2|E2B2, B2E2|B2E2), this means that there are 8 different ways of seating in which E1 would be opposite from B1.

Since there are 3 pairs of elf and bestie, there would be $3 \times 8 = 24$ different ways in which a pair of elf and bestie would be sitting opposite from each other.

If for example, E1 chooses to sit first, then there are $5! = 120$ ways that the rest of the people are going to arrange themselves. $\frac{24}{120} = \frac{1}{5}$.

c)

If E1 were to sit down first, then B1 cannot sit next to E1, therefore there is a $\frac{3}{5}$ probability of not sitting with E1.

There are 3 seats which B1 can sit on to not be next to E1. The remaining cannot be sitting next to each other, so if B1 sits in the middle seat, then there are 4 ways which the remainders can sit to not be next to each other.

If B1 chooses to not sit in the opposite seat, then there is a total of 4 ways which E2 is opposite from B2 to satisfy the diagram. Since there would be 2 pairs, then $4 \times 2 = 8$ ways in which the remaining 4 can be rearranged in this pattern.

This is only for 1 seat in which B1 is not opposite to E1, therefore for the other seat we double this, $8 \times 2 = 16$ ways.

Adding this by the chance that B1 is sitting opposite to E1 then it becomes $4 + 16 = 20$ ways, where E1 and B1 are not next to each other, and neither are the rest.

$\frac{20}{120} = \frac{1}{6}$.

