Is There More to Discover about Four Consecutive Numbers?

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Consecutive defintion:

$a = a$

$b = a+1$

$c = a+2$

$d = a+3$

1. $(a^2+d^2)-(b^2+c^2)$

$= [a^2+(a+3)^2]-[(a+1)^2+(a+2)^2]$

$= (a^2+a^2+6a+9)-(a^2+2a+1+a^2+4a+4)$

$= 2a^2+6a+9-2a^2-6a-5$

$= 4$

$(a^2+d^2)-(b^2+c^2)$ is always equal to 4 because the value is independent of the values of a, b, c, d.

2. $(a^2+b^2+c^2+d^2)-(1+2+3)$

$= a^2+(a+1)^2+(a+2)^2+(a+3)^2-6$

$= a^2+a^2+2a+1+a^2+4a+4+a^2+6a+9-6$

$= 4a^2+12a+8$

$= 4(a^2+3a+2)$

$\therefore [(a^2+b^2+c^2+d^2)-(1+2+3)]$ is divisible by 4.

$4(a^2+3a+2)$

$= 4(a+2)(a+1)$

$\because$ Either $(a+2)$ or $(a+1)$ is even, and the other one is odd. Even means that it is divisible by 2.

$4 \times 2 = 8$

$\therefore [(a^2+b^2+c^2+d^2)-(1+2+3)]$ is divisible by 8.

3. Since abcd is a product of four consecutive numbers, two numbers from a, b, c, d would be a multiple of 2, in which one of them would be a multiple of 4. Also, at least one of the four consecutive numbers is a multiple of 3.

$2 \times 4 \times 3 = 24$

$\therefore$ abcd is divisible by 24.

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4. $\sqrt{abcd+1}$ can be expressed as $\sqrt{24\mathbb{Z}+1}$ because abcd is divisible by 24 as proven previously. I calculated and analysed the different results for $\sqrt{24\mathbb{Z}+1}$ with different integer values of $\mathbb{Z}$ in Excel.

Depending on different values of $\mathbb{Z}$, $\sqrt{24\mathbb{Z}+1}$ may output integers or decimals. The integer results include all primes except 1, 2 and 3, starting from 5. The integer results also include multiples and squares of primes that are bigger than or equal to 5.

I then tried to prove that all prime numbers, except for 1,2 and 3 can be expressed as $\sqrt{24\mathbb{Z}+1}$.

Let p be any prime number bigger than and including 5. We want to prove that:

$$p = \sqrt{24\mathbb{Z}+1}$$

$$\text{So} \quad p^2-1 = 24\mathbb{Z}$$

$$p^2-1$$
$$=(p+1)(p-1)$$

As p is a prime number that is not 1,2 or 3, it has to be odd. So, $(p+1)$ and $(p-1)$ are even, meaning that they are divisible by 2. Also, because $(p+1)$ and $(p-1)$ are two consecutive even numbers, one of them is divisible by 4.

Also, because $(p-1)$, p and $(p+1)$ are three consecutive integers. One of them is a multiple 3. However, p cannot be a multiple of 3 because it is a prime number except from 1, 2, 3. So $(p+1)(p-1)$ is a multiple of 3.

$$2\times4\times3=24$$

$\therefore (p^2-1)$ is a multiple of 24.

$$p^2-1 = 24\mathbb{Z}$$
$$p = \sqrt{24\mathbb{Z}+1}$$