For these questions, because we want a general answer that can apply to all the cases, it's easier for us to work with algebra than a specific number.
Because a, b, c and d are consecutive numbers, it means each number from lowest to highest has a difference of 1. So if we let $a = x$, then we will have $b = x + 1$, $c = x + 2$, $d = x + 3$. And if we know this, then we can solve most of these problems easily.

1.
For the first question, it's asking us what does $(a^2 + d^2) - (b^2 + c^2)$ equal to. We can bring the previous expressions in here.

$[x^2 + (x + 3)^2] - [(x + 1)^2 + (x + 2)^2]$ **And this is the equation we get**

$= (x^2 + x^2 + 6x + 9) - (x^2 + 2x + 1 + x^2 + 4x + 4)$ **We simplify it and get rid of all the brackets**

$= (2x^2 + 6x + 9) - (2x^2 + 6x + 5)$

$= 2x^2 + 6x + 9 - 2x^2 - 6x - 5$

$= \cancel{2x^2} + \cancel{6x} + 9 - \cancel{2x^2} - \cancel{6x} - 5$

$= 9 - 5$

$= 4$

And then we will find out that all the unknowns are being canceled out, what is left is four. So we know that the solution of the equation is 4.

2.
For the second question, we also need to use the algebra method. It's pretty similar to the last question.

$(a^2 + b^2 + c^2 + d^2) - (1 + 2 + 3)$, and we will get

$= [x^2 + (x + 1)^2 + (x + 2)^2 + (x + 3)^2] - (1 + 2 + 3)$ **We also need to simplify this equation first**

$= (x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 + x^2 + 6x + 9) - 6$

$= 4x^2 + 12x + 8$ **And then we will find that the highest common factor is 4 here, so we factorized it of the equation.**

$= 4(x^2 + 3x + 2)$

$= 4(x + 1)(x + 2)$ **Here it's not hard to see that $x^2 + 3x + 2$ can be factorized and becomes $x + 2$ times $x + 1$**

Until here, we can see at least the equation $(a^2 + b^2 + c^2 + d^2) - (1 + 2 + 3)$ is always divisible by 4, but how is it divisible by 8? There is nothing else we can do with this 4, so we have to turn our attention to the second half of the equation: $(x + 1)(x + 2)$. We know that if two numbers multiply together, as long as one of them is an even number, the product is going to be an even number as well. Here, for $(x + 1)(x + 2)$, whatever the number x is, we will always have an even number and an odd number multiply together. So we can always get an even product. All the even numbers can be divided by 2, so it means that $(x + 1)(x + 2)$ is divisible by 2! And we've already know that the whole equation equals $(x + 1)(x + 2)$ times 4, so 2×4=8, and we finally find out that the whole equation is divisible by 8.

3.

For the third question, I started with algebra at the beginning, and it doesn't seems to work that well here. If you also use the method before which we let $a = x, b = x + 1$, $c = x + 2, d = x + 3$ and put it in the equation, simplify it and expand the brackets, we will end up with an equation like this: $x^4 + 6x^3 + 11x^2 + 6x$, maybe you will notice that the factors add up to 24, but that's the only clue I can find here and it's not that useful. So let get back to the question. Four consecutive number multiply together, how is it divisible by 24? We know if a number is divisible by x, it must be divisible by any of the factors of x. So think about 24, we will know that $24 = 2 \times 2 \times 2 \times 3$. First, we know all the even number are divisible by 2, it means every 2 consecutive numbers, there is a number that are divisible by 2, there are 4 consecutive number in total, so there should be 2 even numbers which are divisible by 2. So we solve the 2 here, what's left is $2 \times 2 \times 3$. For being divisible by 3, we know that 3×1=3, 3×2=6, 3×3=9, …, so every three consecutive numbers, there is a number which can is divisible by 3. And we've got 4 consecutive numbers here! So there must be 1 number in it which is divisible by 3. So now we solve 3 as well, what's left is 2×2. Until now, many of you may already know what's going on and what to do here. 2×2 is 4. And every 4 consecutive numbers there is a number that is divisible by 4. And we have exactly 4 consecutive numbers here, so there must be 1 number which is divisible by 4 as well. Now we know that in these four consecutive numbers, we will be able to find all 2, 3, and 4 in the factors of these four numbers. So $abcd$ will be always divisible by 24.

4.

For the forth question, I started with algebra as usual. Let $a = x, b = x + 1, c = x + 2$, $d = x + 3$ and put it into $\sqrt{abcd + 1}$, and we get $\sqrt{x(x + 1)(x + 2)(x + 3) + 1}$.

$$\sqrt{x(x + 1)(x + 2)(x + 3) + 1}$$
$$= \sqrt{x^4 + 6x^3 + 11x^2 + 6x + 1}$$

We also need to simplify this equation first. (We can use the simplified equation from the third question)

Here we have to find a way to factorize the equation. Because we have x^4 so we will know there must be two x^2 multiply together, and because the only constant is 1, so we know it can only be 1×1. And it gives us this equation $(x^2 + ax + 1)(x^2 + bx + 1)$. Now the only thing we have to do is to find what's a and b. And it's not hard to try it a few times on paper to get the answer $a = b = 3$.

So the equation will be: $= \sqrt{(x^2 + 3x + 1)^2}$

$$= \left| x^2 + 3x + 1 \right|$$

Because here we have a square and a root, the square and root of a number is its absolute value.

Now, we totally get rid of the root, so it shows us that $abcd + 1$ is always a perfect root. And because x is an integer, $x^2 + 3x + 1$ will be an integer as well. The solution of $\sqrt{abcd + 1}$ must be an integer.

Is that the end of it? Yes, but not necessarily. Let's go back to the first equation, if we bring some constants into the equation, it will give us something like this:

While	$\sqrt{abcd+1}$	
$a=1$, $b=2$, $c=3$, and $d=4$	$\sqrt{1\times2\times3\times4+1}$	5
$a=2$, $b=3$, $c=4$, and $d=5$	$\sqrt{2\times3\times4\times5+1}$	11
$a=3$, $b=4$, $c=5$, and $d=6$	$\sqrt{3\times4\times5\times6+1}$	19
$a=4$, $b=5$, $c=6$, and $d=7$	$\sqrt{4\times5\times6\times7+1}$	29

We will then find a pattern from it, the solution of $\sqrt{abcd+1}$ is always $ad+1$, but this is only the pattern we find, we still need to prove its generality. Let $a=x$, $b=x+1$, $c=x+2$, $d=x+3$, and bring it into $ad+1$:

$ad+1$

$=x(x+3)+1$

$=x^2+3x+1$

$\because \sqrt{abcd+1}=\left|x^2+3x+1\right|$ and $ad+1=x^2+3x+1$

$\therefore \sqrt{abcd+1}=\left|ad+1\right|$