Continuing to explore the four consecutive numbers challenge.

a, b, c, d – four consecutive numbers

① Why can't $bd - ac$ be even?

Solution: Let's call $b = a+1$, $c = a+2$, $d = a+3$

$$bd = (a+1)(a+3) = a^2 + 4a + 3$$
$$ac = a \times (a+2) = a^2 + 2a$$

$$\Rightarrow bd - ac = a^2 + 4a + 3 - a^2 - 2a =$$
$$= 2a + 3 - \text{odd},$$

because $2a$ is even and 'even' $+3$ is odd

② What is $bc-ad$ always equal to ?

Solution: $b=a+1$, $c=a+2$, $d=a+3$

$$bc=(a+1)(a+2)=a^2+3a+2$$

$$ad=a(a+3)=a^2+3a$$

$$\therefore bc-ad=a^2+3a+2-a^2-3a=2$$

Answer: 2

(3) Why must the sum $a+b+c+d$ have an odd factor ?

Solution: $b=a+1$, $c=a+2$, $d=a+3$

$$a+b+c+d = 4a+6$$

$$4a+6 = 2(2a+3)$$

$2a+3$ is odd no matter what a is. (see (1))

④ Why can't the sum $a+b+c+d$ be a multiple of 4 ?

Solution:

$$a+b+c+d = 4a+6$$

$4a$ is a multiple of 4

$4a+4$ is a next multiple of 4

$4a+8$ is a next multiple of 4

but $4a+6$ is not a multiple of 4 $4a+6=4(a+1.5)$

not an integer

(5) Which consecutive numbers are such that $a+b+c+d$ divides exactly by 3?

Solution:

$a+b+c+d = 4a+6$. If $4a+6$ is divisible by 3, it is equal to $3m$, where m is an integer.

$4a+6=3m$

$4a=3m-6$

$a=\frac{3}{4}m-\frac{6}{4}$

$a=0.75m-1.5$

if $m=6$ $a=4.5-1.5=3$

$m=2$ $a=1.5-1.5=0$

$m=10$ $a=7.5-1.5=6$

$a=3, b=4, c=5, d=6$

$a+b+c+d=18$ $18\div3=6$

$a=0, b=1, c=2, d=3$

$a+b+c+d=6$ $6\div3=2$

$a=6, b=7, c=8, d=9$

$6+7+8+9=30$ $30\div3=10$

M: 2, 6, 10, 14, 18, ..., $4n-2$,

$a=3n-3$

$b=3n-2$

$c=3n-1$

$d=3n$

Answer: numbers, satisfying $a=3n-3, b=3n-2, c=3n-1, d=3n$ where n = integer