H/W

What does it all add up to?

1 3/1/22

5, 6, 7, 8 total = 26
6, 7, 8, 9 total = 30
7, 8, 9, 10 total = 34
0, 1, 2, 3 total = 6
1, 2, 3, 4 total = 10
2, 3, 4, 5 total = 14

Disserence of 4

From these you can see that by moving each number up a step, e.g 1,2,3,4 → 2,3,4,5 the total goes up by 4. This is because the disserence between the sirst number in the sirst set of numbers and the last number in the second set of numbers is 4. E.g 1 and 5 have a disserence of 4.

0, 1, 2, 3 total = 6

$4 \times 1 + 2 = 6$

5, 6, 7, 8 total = 26

$4 \times 6 + 2 = 26$

By multiplying the 2nd number by 4 and adding 2, you get the total.

Using this, we can see if 80 can be created by adding 4 consecutive numbers

Since we don't know the 4 consecutive numbers, let the second one be n.

$80 = 4n + 2$
$78 = 4n$
$19.5 = n$

This isn't a whole number so can't be made by consecutive whole numbers.

This sormula: $4n+2$ consirms that a multiple of 4 can't be made of consecutive integers as it is always a multiple of 2, but not 4 as it is always 2 above a multiple of 4

Extension: 5 consecutive numbers

0, 1, 2, 3, 4 total = 10
1, 2, 3, 4, 5 total = 15
2, 3, 4, 5, 6 total = 20

Those in black circles have a difference of 5 so the total increases by 5

Multiplying these pink circles numbers by 5 gives you the total. Therefore, the total will always be a multiple of 5. If the 3rd number was represented by c, the formula is simply: total = 5c

Extension: 6 consecutive numbers

0, 1, 2, 3, 4, 5 total: 15
1, 2, 3, 4, 5, 6 total: 21
2, 3, 4, 5, 6, 7, total: 27

The black circled numbers have a difference of 6 so the total will increase by 6

Multiplying these by 6 and adding 3 gives you the total. Again, representing the number by c, the formula would be 6c+3. This shows the total won't ever equal a multiple of 6, but rather 3 more (or less).

Challenging Extension

First let's give n a few possible values: 1, 3, 5 and 7

Of course anything that is whole is a multiple of 1

For 3 we should get a few groups of consecutive numbers:
0, 1, 2 = 3
1, 2, 3 = 6 Although we can see these are divisible by 3, we want a formula
2, 3, 4 = 9 just to be certain
If these were n then total = 3n

We've already done 5 so 7 is next
0, 1, 2, 3, 4, 5, 6 21
1, 2, 3, 4, 5, 6, 7 28
2, 3, 4, 5, 6, 7, 8 35
let these be n. total = 7n

I have noticed a pattern where the number you centre your formula around is always found in the middle, at least for an odd number of consecutive numbers.

Using our testing and this pattern, we can say that if n is odd, then the sum of n consecutive numbers is a multiple of n

We are going to do the same for the next statement.

2   0, 1 = 1 $\frac{n}{2} = \frac{2}{2} = 1$
1, 2 = 3
2, 3 = 5
total = 2n + 1   1

These are multiples of 1 not 2.

We've done 4 and 6 and know that the statement holds for them.

8   0, 1, 2, 3, 4, 5, 6, 7 = 28
1, 2, 3, 4, 5, 6, 7, 8 = 36 → Multiples of $\frac{8}{2}$ or 4, but not 8
2, 3, 4, 5, 6, 7, 8, 9 = 44

total = 8n + 4

Another pattern is always that the number the formulas are formed of comes from the total numbers ÷ 2. For 2 consecutive numbers we looked at the first term and so on.

All these formulas are created by the amount of consecutive numbers × n + half of the amount of consecutive numbers. E.g 8 consecutive numbers have a formula of 8n + 4

Using these patterns and the testing, if n is even then the sum of n consecutive numbers is always a multiple of $\frac{n}{2}$ but not n.