1) Show $\phi(15) = 8$

$\phi(15) = n(1, 2, 4, 7, 8, 11, 13, 14) = 8$

#Coprimes

Totient Function

2) $\phi(p)$:

$\phi(p) = p - 1$

$\phi(2) = 1$

$\phi(7) = 6$

$\phi(3) = 2$

$\phi(5) = 4$

Only 2 factors.
One is p itself.
Not coprime.
The rest are as if it is a prime number.
$\therefore \phi(p) = p - 1$

3) $\phi(2^n)$

$\phi(2^1) = 1$
$\phi(2^2) = 2$
$\phi(2^3) = 4$
$\phi(2^4) = 8$

n	1	2	3	4
2^n	2	4	8	16
$\phi(2^n)$	1	2	4	8

$\div 2$ or $\times \frac{1}{2}$

$\phi(2^n) = \frac{2^n}{2}$

$\frac{2^n}{2^1} = 2^{n-1}$

$1 \times 2^{n-1}$ $\phi(p^n)$ $2 \times 3^{n-1}$

$p-1$ $p-1$

$(p-1)(p^{n-1}) =$

$p^n - p^{n-1}$

n	1	2
3^n	3	9
$\phi(3^n)$	2	6

$\phi(3^n) =$

$\frac{2 \times 3^n}{3^1} = 2$

3) $\phi(2^n)$ $\phi(3^n)$

$\phi(2^1)=1$ $\phi(3^1)=2$
$\phi(2^2)=2$ (×2) $\phi(3^2)=6$ (×3)
$\phi(2^3)=4$ (×2) $\phi(3^3)=18$ (×3)
$\phi(2^4)=8$ (×2) $\phi(3^4)=54$ (×3)

n	1	2	3	4
2^n	2	4	8	16
$\phi(2^n)$	1	2	4	8

(÷2 or ×$\frac{1}{2}$)

n	1	2	3	4
3^n	3	9	27	81
$\phi(3^n)$	2	6	18	54

(÷1.5 or ×$\frac{2}{3}$)

$\phi(2^n)=\frac{2^n}{2}$

$\phi(3^n)=\frac{2\times3^n}{3}$

$\frac{2^n}{2^1}=2^{n-1}$

$\frac{2\times3^n}{3^1}=2\times3^{n-1}$

$1\times2^{n-1}$ $\phi(p^n)$ $2\times3^{n-1}$

$p-1$ $p-1$

$(p-1)(p^{n-1})=$

p^n-p^{n-1}

4) $\phi(mn) = \phi(m)\phi(n)$

ONLY when 'n' and 'm' are coprime factors of the value of 'n x m':

e.g1/

$\phi(15) = \phi(3) \times \phi(5)$

Factors of 15:

1 - 15

3 - 5 ← co-prime factors

$\phi(3) = 2$ and $\phi(5) = 4$

$2 \times 4 = 8$ (or $\phi 15$)

e.g2/

$\phi(18) = 6$

Factors of 18:

1 — 18

2 - 9 ← co-prime

3 - 6 ← Not co-prime

$\phi 2 = 1$ (as it is prime) and $\phi 9 = 6$

$1 \times 6 = 6$

(or $\phi(18)$)

When not co-prime:

$\phi 3 = 2$ $\phi 6 = 2$

$(\phi 3) \times (\phi 6)$ is not 6

5) $\phi(n) =$

if prime $= n-1$

if is a multiple of a prime (p) $= n \times \frac{p-1}{p}$

continue until all prime factors of n are exhausted
(if there are multiple of one prime factor, only use it once)