1. $\phi 15 = \phi(3 \times 5) = 8$

1 2 $\cancel{3}$ 4 $\cancel{5}$ $\cancel{6}$ 7 8 $\cancel{9}$ $\cancel{10}$ 11 $\cancel{12}$ 13 14 $\cancel{7}$

2. $\phi p = p - 1$ because no factors are shared as p is its only prime factor

3. $\phi(2n)$ $\phi(2 \times 1) = 1$

$\phi(2 \times 2) = \phi 4 = 2$

$\phi(2 \times 3) = \phi 6 = 2$

$\phi(2 \times 4) = \phi 8 = 4$

$\phi(2 \times 5) = \phi 10 = 4$

$\phi(2 \times 6) = \phi 12 = 4$

$\phi(2 \times 7) = \phi 14 = 6$

$\phi(2 \times 8) = \phi 16 = 8$

$\phi(2 \times 9) = \phi 18 = 6$

$\phi(2 \times 10) = \phi 20 = 8$

where n is even

$\phi 2n = 2\phi n$

As even numbers aren't co-prime to n, they aren't co-prime to $\phi 2n$. However, twice the amount of co-primes are within $2n$ than n because $2n$ has no <u>new</u> prime factors.

where n is odd

$\phi 2n = \phi n$

As 2 is a new prime factor, every 2^{nd} (even) co-prime to n is not coprime to $2n$. $2n$ has twice as many odd co-primes to n $\therefore 2\left(\frac{\phi n}{2}\right) = \phi n$

4. $\phi 3 = 2 \quad \phi 5 = 4 \quad \phi 3 \times \phi 5 = \phi 15$

$\phi n \times \phi m = \phi nm$

where n and m are prime

$(n-1)(m-1) = nm - n - m + 1$

$\phi mn = mn - \left(\underbrace{\frac{mn}{n}}_{\text{multiples of } n} + \underbrace{\frac{mn}{m}}_{\text{multiples of } m} - 1\right) \rightarrow 1$, which is accounted for twice.

$\phi n \times \phi m = \phi nm$ when n and m are co-prime because nm must have the same amount of different prime factors as ~~the sums~~ ~~m~~ prime factors of n and prime factors of m added together.

5. $\phi n = n - \frac{n}{p1} - \frac{n}{p2} + \frac{n}{p1p2} - \frac{n}{p3} + \frac{n}{p1p3} + \frac{n}{p2p3} - \frac{n}{p4} + \frac{n}{p1p4} + \frac{n}{p2p4} \ldots$

where $p_1, p_2 \ldots p_k$ are different prime factors of n

For each p, subtract $\frac{n}{pk}$ and add n divided by ~~every~~ every combination of ~~previous~~ two ~~previous~~ prime factors including ~~$\frac{n}{p}$~~ pk $\left(+\frac{n}{p1pk} + \frac{n}{p2pk} \ldots + \frac{n}{pk-1pk}\right)$

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May 2021.