DIGITAL EQUATION

$k=0 \Rightarrow 0(10-0) = 0$ div by 3

$k=1 \Rightarrow 1(10-1) = 9$ div by 3

$k=2 \Rightarrow 2(10-2) = 16$

$k=3 \Rightarrow 3(10-3) = 21$ div by 3

$k=4 \Rightarrow 4(10-4) = 24$ div by 3

$k=5 \Rightarrow 5(10-5) = 25$

$k=6 \Rightarrow 6(10-6) = 24$ div by 3

$k=7 \Rightarrow 7(10-7) = 21$ div by 3

$k=8 \Rightarrow 8(10-8) = 16$

$k=9 \Rightarrow 9(10-9) = 9$ div by 3

$(k)(k-1)(k+1)$ is the product of three consecutive numbers

Out of three consecutive numbers, at least one will be divisible by 3

$\therefore$ Entire product is divisible by 3

$N = 100a + 10b + c$

$S = a + b^2 + c^3$

$$100a + 10b + c = a + b^2 + c^3$$

$$99a + 10b - b^2 = c^3 - c$$

$$99a + 10b - b^2 = c(c^2-1)$$

$$99a + 10b - b^2 = c(c-1)(c+1)$$

RHS is divisible by 3

$\therefore$ LHS must be div. by 3 too

$3 \mid c(c-1)(c+1)$

$3 \mid 99a + 10b - b^2$

$3 \mid b(10-b)$

This is equivalent to first part of the question to figure out $10k - k^2$

To be div. by 3, b should be either <u>0, 1, 3, 4, 6, 7, 9</u>

It can also be seen that $c^3 - c$ will be greater than or equal to 99 since $a \geq 1$ and $b \in (0, 1, 2, 3, \ldots 9)$

All values of $c^3 - c$ are 0, 6, 24, 60, 120, 210, 336, 504, 720

Only values for $c \geq 5$ are $c^3 - c$ greater than 99.

$99a \in \{99, 198, 297, 396, 495, 594, 693, 792, 891, \cancel{990}\}$

$10b - b^2 \in \{0, 9, 21, 24\}$

$c^3 - c \in \{120, 210, 336, 504, 720\}$

a, b, c are independant of each other

$\underline{99a + 10b - b^2} = \underline{c^3 - c}$

Max = 891+24 Max = 720

$\therefore$ 99a can't be 891, 792,

Now, it's brute force and educated guess for all values

120 can be 99 + 21 only ⇒ 1

210 can be ~~99 + 111~~ ~~198 + 22~~ none ⇒ 0

336 can be ~~297 + 39~~ none ⇒ 0

504 can be 495 + 9 1 case ⇒ 1

720 can be none ⇒ 0

(1) $c^3 - c$ is 120 at $\underline{c=5}$

$99a$ is 99 at $\underline{a=1}$

$10b - b^2$ is 21 at $\underline{b=3}$ and $\underline{b=7}$

(2) $c^3 - c$ is 504 at $\underline{c=8}$

$99a$ is 495 at $\underline{a=5}$

$10b - b^2$ is 9 at $\underline{b=1}$, and $\underline{b=9}$

Thus from all cases

1, 3, 5 $\Rightarrow$ 135
1, 7, 5 $\Rightarrow$ 175
5, 1, 8 $\Rightarrow$ 518
5, 9, 8 $\Rightarrow$ 598