# Terminating or not

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## **Critical observations**

Factorising the fractions from the problem into prime powers, we can get the following:

$$\frac{2}{3} \quad \frac{4}{5} \quad \frac{17}{50} \quad \frac{3}{16} \quad \frac{7}{12} \quad \frac{5}{8} \quad \frac{11}{14} \quad \frac{8}{15}$$

$$\frac{2}{3} \quad \frac{2^2}{5} \quad \frac{17}{2 \times 5^2} \quad \frac{3}{2^4} \quad \frac{7}{2^2 \times 3} \quad \frac{5}{2^3} \quad \frac{11}{2 \times 7} \quad \frac{2^3}{3 \times 5}$$

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The Orange Fractions terminate, the rest do not. We can also observe that the denominator of these Orange Fractions fractions includes powers of 2 or 5 only. The non-terminating fractions also include powers of 2 or 5, but they also include powers of other primes such as 3 or (and) 7

We know that any number of the form: $\frac{p}{10^q}$ terminates. E.g. $\frac{4}{10^2} = 0.04$ and $\frac{4}{10^{-2}} = 40$. This is because $10^q$ constitutes shifting the decimal point. The fraction form itself can be broken down to $\frac{p}{2^q \cdot 5^q}$ This also shows why only Orange Fractions terminate and the rest do not.

## More examples

Let us work with an example and then generalise it to any fraction.

Let the fraction be: $\dfrac{8281}{33280}$

$$\begin{aligned}\frac{8281}{33280} &= \frac{7^2 \cdot 13^2}{2^9 \cdot 13 \cdot 5} \\ &= \frac{7^2 \cdot 13}{2^9 \cdot 5}\end{aligned}$$

This fraction is thus terminating, because the denominator only has powers of 2 and 5.

We can also verify that it is indeed terminating

$$\frac{7^2 \cdot 13}{2^9 \cdot 5} = 0.6220703125$$

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Let another fraction be: $\dfrac{8281}{99840}$

$$\begin{aligned}\frac{8281}{99840} &= \frac{7^2 \cdot 13^2}{2^9 \cdot 3 \cdot 13 \cdot 5} \\ &= \frac{7^2 \cdot 13}{2^9 \cdot 3 \cdot 5}\end{aligned}$$

This fraction is not terminating, because the denominator has powers of 2 and 5, but also of 3

$$\frac{7^2 \cdot 13}{2^9 \cdot 3 \cdot 5} = 0.082942708\dot{3}$$

# How to identify if a fraction terminates? - Generalised strategy

We can let a general fraction be $\frac{p}{q}$

STEP 1: Factor out the numerator and denominator into its prime factors.

$$\frac{p}{q} = \frac{2^{p_2} \cdot 3^{p_3} \cdot 5^{p_5} \cdots}{2^{q_2} \cdot 3^{q_3} \cdot 5^{q_5} \cdots}$$

NOTE: $p_i$ is the power of prime factor $i$ in the numerator. Similarly, $q_i$ denotes power of prime factor $i$in the denominator

NOTE 2: $p_i$ or $q_i$ can be 0

STEP 2: Observe the powers of prime factors. In particular, whether any $q_i > p_i \quad i \notin 2{,}5$. We can do this by finding $q_i - p_i$. If this value is negative, $q_i \not> p_i$

$$\frac{p}{q} = \frac{2^{p_2} \cdot 3^{p_3} \cdot 5^{p_5} \cdots}{2^{q_2} \cdot 3^{q_3} \cdot 5^{q_5} \cdots}$$

STEP 3: If for some $i$, $q_i > p_i \quad i \notin 2{,}5$, then the fraction is non-terminating because it contains another prime factor other than 2 or 5.

If this condition is false for every prime, then the fraction is terminating.