



In a "7-sandwich", how many red squares are covered and how many blue squares are covered?

There are 7 red squares covered and 7 blue squares covered.

If it were possible to make a "6-sandwich", how many red squares and how many blue squares would be covered?

For a 6-sandwich, total there are 12 squares covered. 6 are red, 6 are blue.

If you place a 1 on a blue square, on which colour will you place the other 1?

Blue

If you place a 2 on a blue square, on which colour will you place the other 2?

Red

If you place a 3 on a blue square, on which colour will you place the other 3?...

Blue

In general, what can you say about the colours on which you place pairs of numbers?

If it is an odd number, it covers the same colour of tile. e.g. Red-Red, Blue-Blue

If it is an even number, it covers different colours of tile. e.g. Red-Blue, Blue-Red

When you try to make a sandwich with the numbers from 1 to 5, or from 1 to 6, what goes wrong?

For a 6-sandwich, in total there should be 12 tiles covered. 6 are red tiles, 6 are blue tiles.

We try:

Number	Possible Colours of Tiles Covered
1	R-R
2	R-B
3	R-R
4	R-B
5	B-B

6	IMPOSSIBLE!
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It is impossible because in 6-sandwich there are 3 even numbers which cover 3 R and 3 B. Odd numbers can only cover an even number of R or B. They cannot make another 3R or 3B.

For a 5-sandwich, in total there should be 10 tiles covered. 5 are red tiles, 5 are blue tiles. We try:

Number	Possible Colours
1	R-R
2	R-B
3	B-B
4	R-B
5	IMPOSSIBLE!

It is impossible because in 5-sandwich there are 2 even numbers which cover 2 R and 2 B. Odd numbers can only cover an even number of R or B. They cannot make another 3R or 3B.

Which other sandwiches are impossible? How can you be sure?

Odd numbers can only cover an even number of R or B.

Number Sandwich	Number of Red and Blue Need to be Covered	Even numbers in the sandwich	No. of R and B covered by even number	No. of R and B needed to be covered by odd numbers	POSSIBLE?
1					NO
2	2R, 2B	2	1R, 1B	1R, 1B	NO
3	3R, 3B	2	1R, 1B	2R, 2B	YES
4	4R, 4B	2, 4	2R, 2B	2R, 2B	YES
5	5R, 5B	2, 4	2R, 2B	3R, 3B	NO
6	6R, 6B	2, 4, 6	3R, 3B	3R, 3B	NO
7	7R, 7B	2, 4, 6	3R, 3B	4R, 4B	YES
8	8R, 8B	2, 4, 6, 8	4R, 4B	4R, 4B	YES

9	9R, 9B	2, 4, 6, 8	4R, 4B	5R, 5B	NO
10	10R, 10B	2, 4, 6, 8, 10	5R, 5B	5R, 5B	NO
11	11R, 11B	2, 4, 6, 8, 10	5R, 5B	6R, 6B	YES
12	12R, 12B	2, 4, 6, 8, 10, 12	6R, 6B	6R, 6B	YES
13	13R, 13B	2, 4, 6, 8, 10, 12	6R, 6B	7R, 7B	NO
14	14R, 14B	2, 4, 6, 8, 10, 12, 14	7R, 7B	7R, 7B	NO
15	15R, 15B	2, 4, 6, 8, 10, 12, 14	7R, 7B	8R, 8B	YES

We can see a pattern:

If the number sandwich is an even number. When divided by 2, if the quotient is odd number, then it is impossible to make the number sandwich.

If the number sandwich is an odd number. When added by 1 and then divided by 2, if the quotient is odd number, then it is impossible to make the number sandwich.