SATISFYING STATEMENTS

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I think it's easy to find a number, when we consider just two statements. We can even guess it (for example 15). So I am going to find a general solution to situations when we consider three or four statements.

1 First, look at the statements:

1. "Multiples of five"
2. "Triangular numbers"
3. "Even, but not multiples of four"
4. "Multiples of three but not multiples of nine"

2 If we use mathematical language:

1. $5a$
2. $\frac{b \cdot (b+1)}{2}$
3. $4x - 2$
4. We have two expressions here:
   (a) $9y_1 - 6$
   (b) $9y_2 - 3$

3 Let's calculate this ...

1. $5a$
2. $\frac{b \cdot (b+1)}{2}$
3. $2(2x - 1)$
4. We can write both expressions as $3 \cdot (\mathit{something})$ – this is important.
   (a) $3(3y_1 - 2)$
   (b) $3(3y_2 - 1)$

4 Look at the statements 1, 3 and 4.

We see, that the number we are looking for is certainly a multiple of $5 \cdot 2 \cdot 3 = 30$. If we consider just these three statements, the answer is simple – all multiples of 30 are solutions to these three statements – but ATTENTION. If we multiply 30 by 2 or 3 or every number, which is a multiple of 2 or 3, then the statements 3 or 4 will not be considered. So solutions are only the numbers formed like $30p$, where p is a prime number or a product of two prime numbers except 2 and 3.

EXAMPLE SOLUTIONS: 150, 210, 330, 390, 510 ...

5 What about statement 2?

So we must equate general solution of three statements above and STATEMENT 2 – $\frac{b \cdot (b+1)}{2}$

$$30p = \frac{b \cdot (b+1)}{2}$$

$$60p = b \cdot (b+1)$$

Now we have to find factors of number 30.
$\mathscr{D}_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$$\begin{array}{ccc} 1 & \cdot & 30 \\ 2 & \cdot & 15 \\ 3 & \cdot & 10 \\ 5 & \cdot & 6 \end{array}$$

Then write predecessors and successors of factors (green). We are going to find a solution, when one of these successors and predecessors has a form like that: $2p$ (p means prime number) because of $\frac{b(b+1)}{2}$.

$$\begin{array}{ccc} {}_{0}\,1\,{}_{2} & \cdot & {}_{29}\,30\,{}_{31} \\ {}_{1}\,2\,{}_{3} & \cdot & {}_{14}\,15\,{}_{16} \\ {}_{2}\,3\,{}_{4} & \cdot & {}_{9}\,10\,{}_{11} \\ {}_{4}\,5\,{}_{6} & \cdot & {}_{5}\,6\,{}_{7} \end{array}$$

Number 14 is the only appropriate number. We can write it as $2p = 2 \cdot 7$. If we multiply 30 by 7, we get 210 – this is the solution.