

The question was nebulous at first, but if you read it properly, it does become clearer. 99% of people who have measles have spots, or $\frac{99}{100}$ people with measles get spots, so logically $\frac{1}{100}$ people with measles don't get spots. Ben has spots, are these measles?

Well it depends. It depends on his symptoms. Not all red spots are measles. Measles are only one of many conditions where you (a) get spots and/or (b) get similar spots (question does not specify).

The question tells us that there are 12 million children in the UK of whom 10% get spots or 1,200,000, but only 1,200 kids get measles (0.01%). It's unlikely to be measles, purely based on the numbers, but it does depend on his other symptoms.

We can put this information in a box, then work out the blanks (just subtract to find population without spots, and population without measles). I then calculated 99% of the children with measles, to find the number with measles AND spots. Simple subtraction then filled in the rest of the blanks.

	Measles	Not Measles	Total Population
Spots	1188 (99% of 1,200)	1,198,812 (1,200,000 – 1188)	1,200,000
No spots	12 (1% of 1,200)	10,799,988 (10,800,000 – 12)	10,800,000
Total Population	1,200	11,998,800	12,000,000

Now that I have some numbers to work with, I can try to answer the question:

$$\text{Probability} = \frac{\text{number of desired/successful outcomes}}{\text{total number of outcomes}}$$

The probability of getting measles as a child is $\frac{1200}{12000000} = 0.0001$ or 0.01%

Probability works on a scale from 0 (no chance) to 1 (dead cert). A probability of 0.5, indicates an even chance of something happening (e.g. coin flip, of a fair coin).

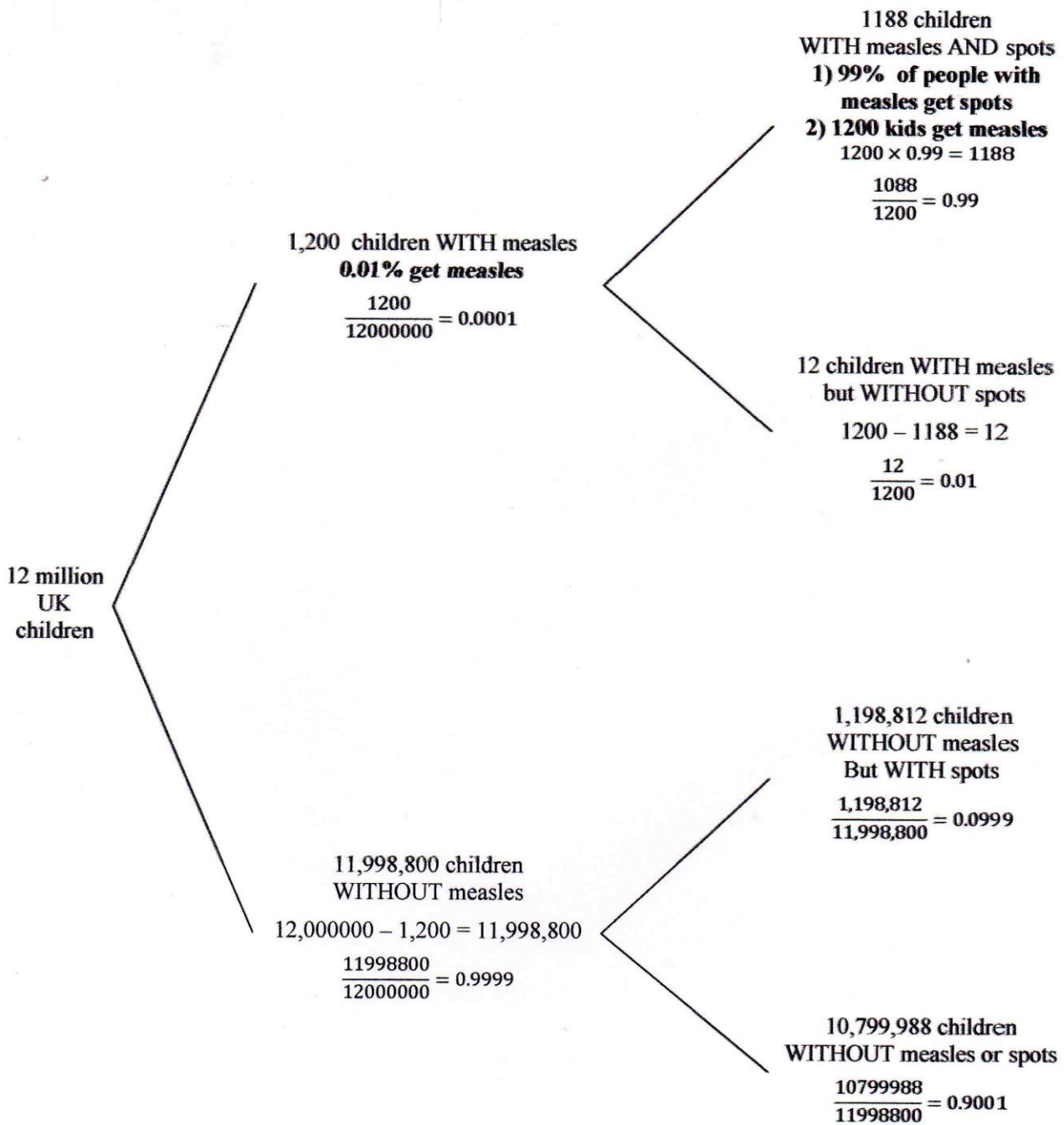
Against that scale, a probability of 0.0001 is very low. This could be due to a number of factors, but is most likely influenced by the national vaccination scheme to prevent measles.

Ben has spots, so he belongs to the population with spots (1,200,000) of which only 1188 have measles. So I can calculate his probability of having measles to be $\frac{1188}{1200000} = 0.00099$, or 0.099% which is slightly less than the probability of having measles in a general population. This is because some children have measles, but no spots. The probability of Ben NOT having measles, is much higher at $1 - 0.00099 = 0.99901$, or 99.901%. This can also be calculated as $\frac{1198812}{1200000} = 0.99901$.

It is therefore highly unlikely that Ben has measles, just because he has red spots, but that does not mean it is impossible, just not probable.

I know that if you get measles, you usually get a very high temperature, and often you get a cold a few days before. I wanted to try and factor this in, but I just couldn't find any data for UK children on the internet (sorry, I really did try). But I did find some interesting data from America, that talks about outbreaks in California, linked to holiday parks that receive high numbers of visitors, and unvaccinated groups within the population <http://www.cdc.gov/measles/cases-outbreaks.html>

You can display the information on a probability tree:



To calculate the probability of a child getting measles (0.0001) and spots (0.99), combine the probabilities:

$$0.0001 \times 0.99 = 0.000099 = 9.9 \times 10^{-5}$$

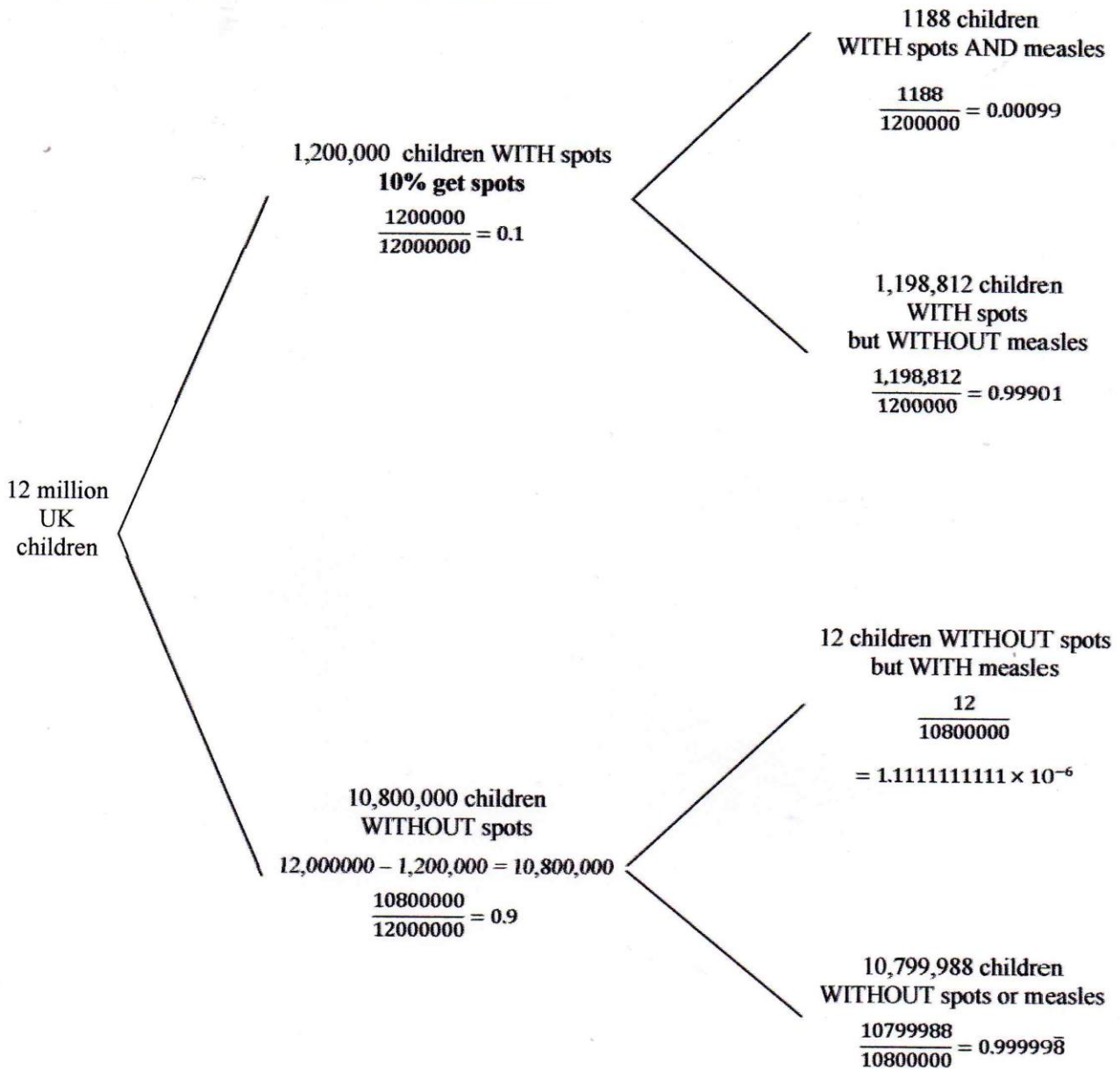
You can then multiply this by the sample size to calculate the population at risk:

$$9.9 \times 10^{-5}(12,000,000) = 1188$$

You can follow the same logic to calculate the probability of a child getting measles WITHOUT spots:

$$0.0001 \times 0.01 = 1 \times 10^{-6}(12,000,000) = 12$$

I followed exactly the same process for the population without measles, but the data was harder to interpret, so I redrew the diagram, using information we already know:



So now I can calculate the probability of a child in the UK of not having spots (0.9) and measles (0.99999 $\bar{8}$) as:

$$0.9 \times 0.99999\bar{8} = 0.899999$$

And if you multiply this by the sample size:

$$0.899999 \times 12000000 = 10,799,988$$

Feedback:

Sorry I wasn't able to do more, I've been doing a lot of piano, and had an exam in November. I did try to solve your tough nuts. To be honest, the probability stuff was really quite tough, it's a different type of maths that obviously I've never come across before (I'm 9 – they've only just started on cube numbers in class...), but I enjoyed reading about this. Please say thank you to the Professor.