

$$\textcircled{1} \quad \frac{6}{6-x} = \frac{3}{x}$$

$$\cdot \quad 6x = 18 - 3x$$

$$9x = 18$$

$$x = 2$$

side of square: 2

$$\textcircled{2} \quad \sqrt{x^2 + (6-x)^2} + \sqrt{x^2 + (3-x)^2} = \sqrt{6^2 + 3^2}$$

$$\sqrt{2x^2 - 12x + 36} + \sqrt{2x^2 - 6x + 9} = \sqrt{45}$$

$$\sqrt{2x^2 - 12x + 36} = \sqrt{45} - \sqrt{2x^2 - 6x + 9}$$

$$2x^2 - 12x + 36 = 45 + 2x^2 - 6x + 9 - 2\sqrt{45} \cdot \sqrt{2x^2 - 6x + 9}$$

$$2\sqrt{45} \cdot \sqrt{2x^2 - 6x + 9} = 6x + 18$$

$$\sqrt{45} \cdot \sqrt{2x^2 - 6x + 9} = 3x + 9$$

$$45(2x^2 - 6x + 9) = 9x^2 + 54x + 81$$

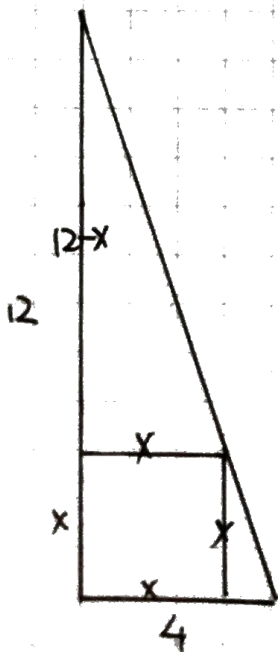
$$5(2x^2 - 6x + 9) = x^2 + 6x + 9$$

$$10x^2 - 30x + 45 = x^2 + 6x + 9$$

$$9x^2 - 36x + 36 = 0$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0 \quad \Rightarrow \quad x = 2$$



$$\frac{12-x}{12} = \frac{x}{4}$$

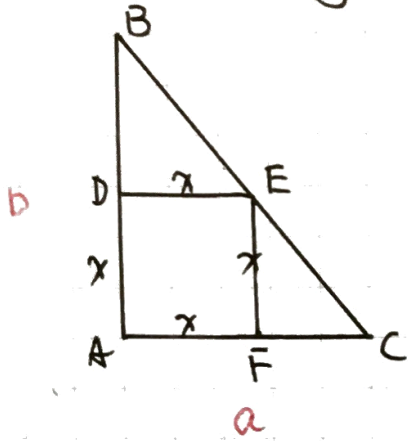
$$12x = 48 - 4x$$

$$16x = 48$$

$$x = 3$$

Method 1

Similar triangle



make the side of the square x

$$\therefore AD = DE = EF = AF$$

$\triangle BDE \sim \triangle BAC$

$$\therefore \frac{BD}{BA} = \frac{DE}{AC} \Rightarrow \frac{b-x}{b} = \frac{x}{a}$$

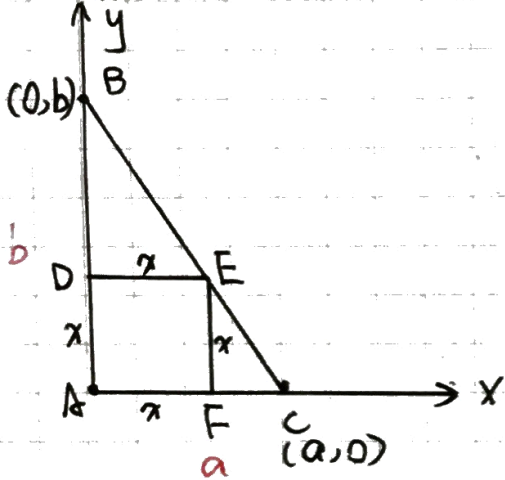
$$ab - ax = bx$$

$$ab = bx + ax$$

$$ab = (a+b)x$$

$$x = \frac{ab}{a+b}$$

Method 2



make the side of the square x

$$y = kx + b$$

$$0 = ak + b$$

$$k = -\frac{b}{a}$$

$$BC: y_1 = -\frac{b}{a}x + b$$

$$E(x, x)$$

$$AE: y_2 = x$$

AE intersects with BC

$$y_1 = y_2$$

$$-\frac{b}{a}x + b = x$$

$$b = \frac{b}{a}x + x$$

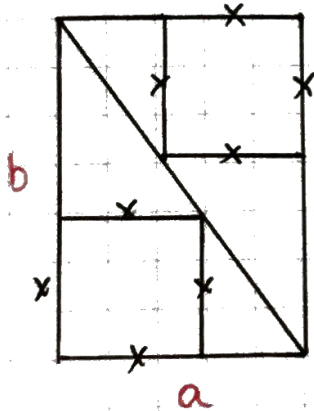
$$\left(\frac{b}{a} + 1\right)x = b$$

$$x = \frac{b}{\frac{b}{a} + 1}$$

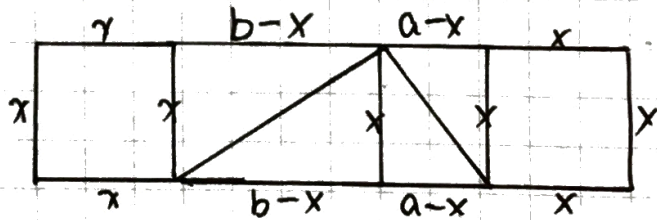
$$x = \frac{ab}{b+a}$$

Method 3

make the side of square x



$$\text{Area}_1 = ab$$



$$\text{Area}_2 = 2x^2 + 2 \cdot \frac{1}{2} \cdot x \cdot (b-x) + 2 \cdot \frac{1}{2} \cdot x \cdot (a-x)$$

$$= 2x^2 + x(b-x) + x(a-x)$$

$$= 2x^2 + bx - x^2 + ax - x^2$$

$$= ax + bx$$

$$\text{Area}_1 = \text{Area}_2$$

$$ab = ax + bx$$

$$(a+b)x = ab$$

$$x = \frac{ab}{a+b}$$