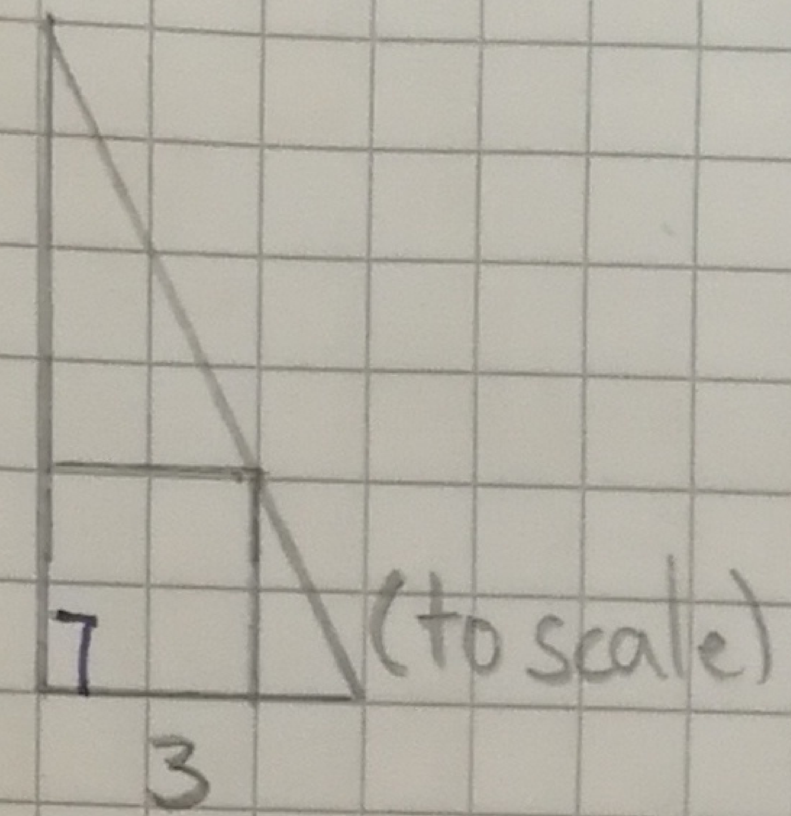
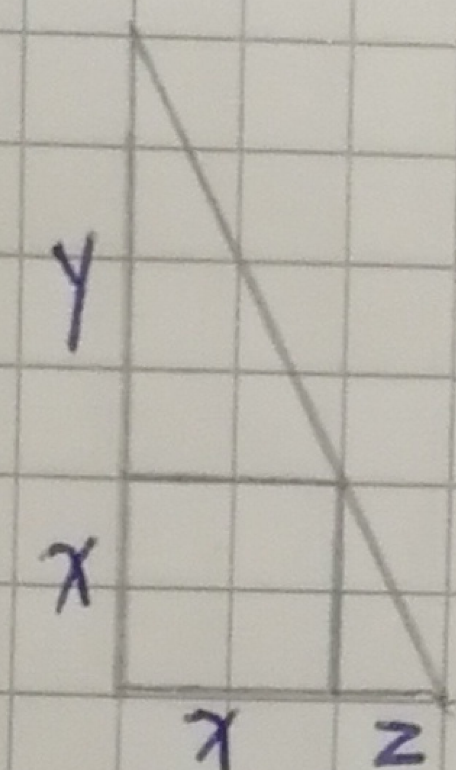
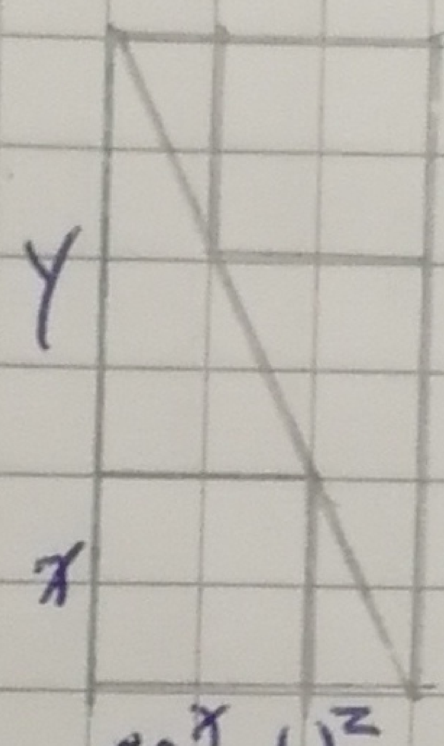
The Square Under the Hypotenuse (find the side length of the square)



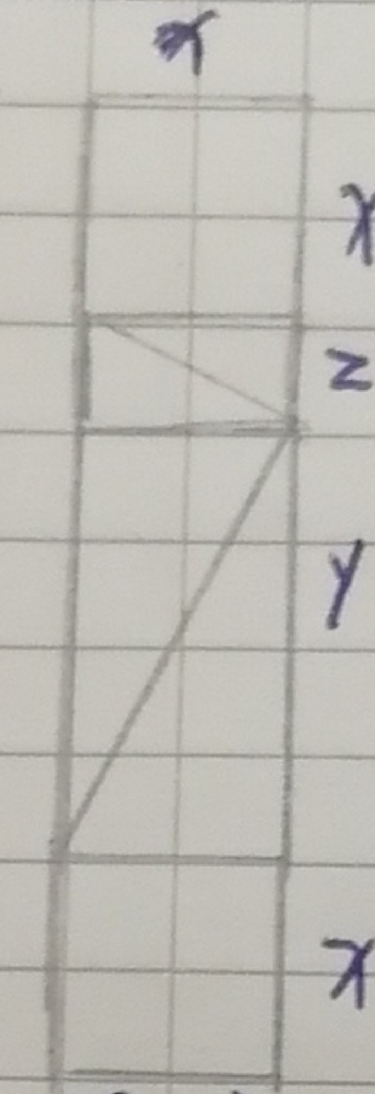
Aside from visual proof, there is also algebraic proof



(fig D)



(fig 1)



(fig 2)

As both fig. 1 and fig. 2 use the same shapes, they have the same area. I will be refering to each ~~equation~~ as "A" followed by the figure number

$A_1 = (x+y)(x+z)$

$A_2 = x(2x+y+z)$

$(x+y)(x+z) = x(2x+y+z)$ [Equation continued right]

$(x+y)(x+z) = x(2x+y+z)$

$x^2+xy+xz+yz = 2x^2+xy+xz$

$x^2+yz = 2x^2$

$yz = x^2$

$y+x=6$

$z+x=3$

$y+x=2z+2x$

$y = x+2z$

$0 = x+2z-y$

$$\left.\begin{aligned} x^2+xy+xz+yz &= 6\times3=18 \\ x^2-yz &= 0 \\ x+2z-y &= 0 \end{aligned}\right\} 2x^2+xy+xz=18$$

$2x^2+xy+xz=18$

$x(2x+y+z)=18$

$2x+y+z = 18\div x$

$2x+y+z = 18\div x$

$x-y+2z = 0$

$3x+3z = 18\div x$

$3x(x+z)=18$

$3x(x+z) = (x+y)(x+z)$

$3x = x+y$

$2x = y$

(see next column)

$2x=y$

$x+y=6$

$2x-y=0$

$3x=6$

$x=2$

$x+y=6$

$y=4$

$x+z=3$

$z=1$

The side length of the square is 2 units

$x:y:z = 2:4:1$

If the side lengths of the triangle were a and b, the side length of the square would be $\frac{1}{3}a$, or conversly, $\frac{2}{3}b$