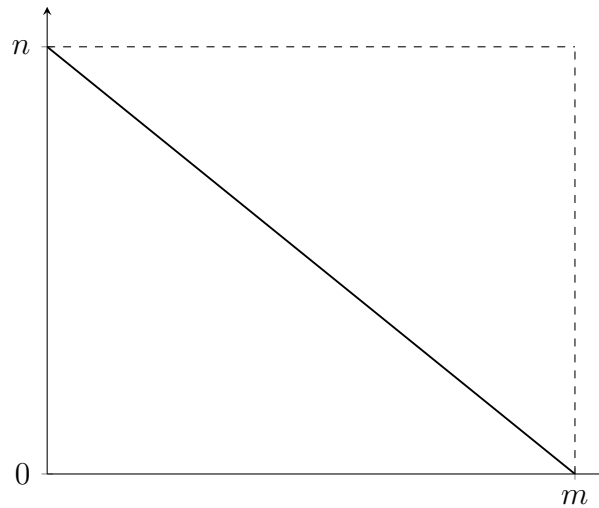


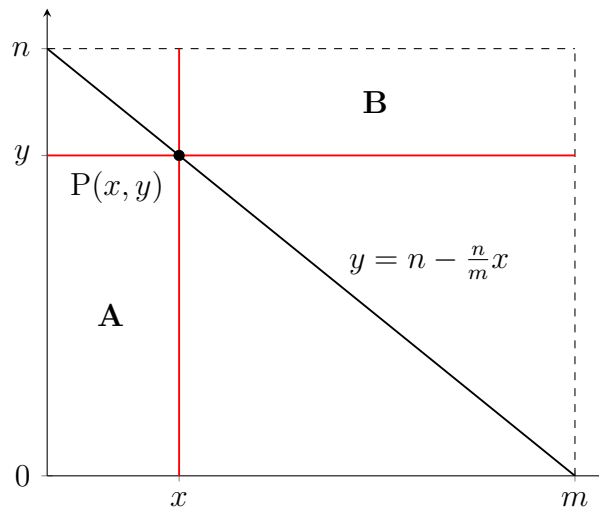
We can begin by drawing a graph containing the rectangle with its bottom left corner placed on the origin, and naming the width and height of the rectangle  $m$  and  $n$ . Drawing on the diagonal of the rectangle, we obtain the following diagram:



We can see that the diagonal contains the points  $(m,0)$  and  $(0,n)$ . Therefore, we can find the gradient of this line, which is  $-\frac{n}{m}$ . Using  $y = mx + c$  with ' $m$ ' =  $-\frac{n}{m}$  and ' $c$ ' =  $n$ , we obtain the equation

$$y = n - \frac{n}{m}x. \quad (1)$$

We can label a general point P on this line, with coordinates  $(x, y)$ , and draw out the rectangles A and B from this point.

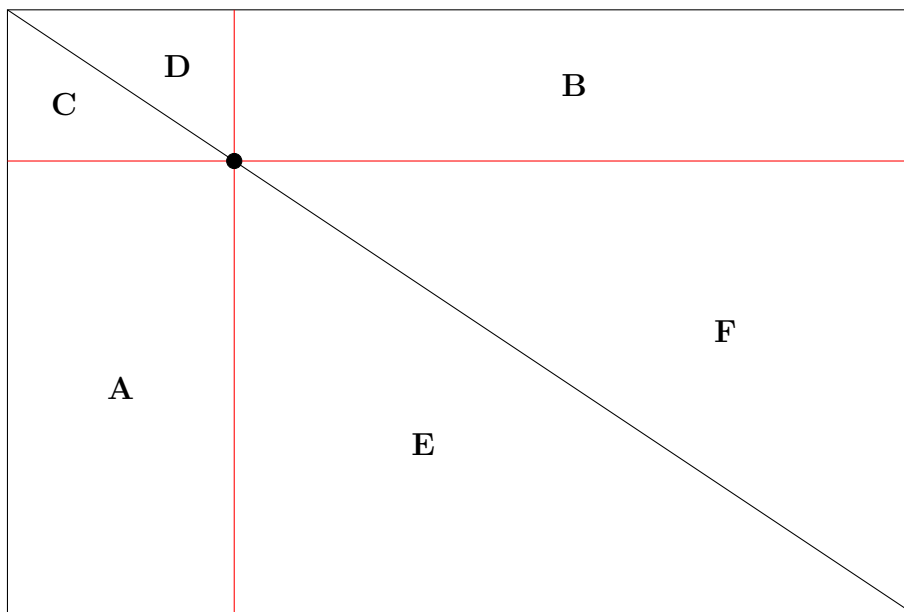


Now, we can see that the area of rectangle A is given by  $A = xy$  and the area of rectangle B by  $B = (m - x)(n - y)$  from the diagram. As we know that  $y = n - \frac{n}{m}x$  for every point  $P(x, y)$  on the diagonal, we can substitute equation (1) in the expressions for the areas of the rectangles. By doing this, we get  $A = x(n - \frac{n}{m}x)$  and doing the same for B and then simplifying gives:

$$\begin{aligned} B &= (m - x)(n - (n - \frac{n}{m}x)) \\ &= \frac{n}{m}x(m - x) \\ &= x(n - \frac{n}{m}x) = A \end{aligned}$$

Therefore, the answer to the problem is that the two areas are equal, no matter which point P on the diagonal you choose, and no matter what the dimensions of the big rectangle are.

Another (arguably simpler) way of solving this problem is to split up the rectangles into triangles, as shown below. You can then use the fact that the two triangles formed when splitting a rectangle by its diagonal have an equal area (due to the triangles having the same base and height when one is rotated). Therefore, you can say that the areas of triangles C and D and triangles E and F are equal.



Therefore, if we denote the areas of the regions above by their respective letters, we obtain the equations:

$$C = D \tag{1}$$

$$E = F \tag{2}$$

And by adding equations (1) and (2) together, we can obtain equation (3) below:

$$C + E = D + F \tag{3}$$

Finally, by considering the whole rectangle being split, we can see equation (4) below, and obtain equation (5) by subtracting equation (3) from equation (4).

$$A + C + E = B + D + F \tag{4}$$

$$A = B \tag{5}$$

Therefore, we can conclude that for every rectangle and every point P on the diagonal of that rectangle, that the area of rectangles A and B are always equal.