K Factorizing Quadratics – Generalizations

Question 3 – Finding the different values of *b* with a fixed value of c

Given that x^2 +bx+c factorizes to (x+*r*)(x+*s*), and *r* • *s* = c, we can say that *r* and *s* are factor pairs of c. We also know that the coefficient b is derived from the sum of the pair that multiplies to form c. The amount of different values of b, depends on the factor pairs of c. Therefore,

Amount of factor pairs of c = Amount of different values for b

Example:

x^2 +bx+10
The factor pairs of 10 are (5,2), (-2,-5), (10,1) and (-1,-10). We can add each of these 4 pairs: 7, -7, 11 and -11. As there are 4 different factor pairs of 10, there are 6 different values for b.

Question 5 – Finding the different values of *b* with fixed values of a and c

The difference between this question and the previous one is that this question introduces a value for a. To solve this, we use the same principle of factor pairs, but instead of factor pairs of a, we use factor pairs of ac.

Amount of factor pairs of ac = Amount of different values for b

Example:

$-2x^2$+bx+6
The factor pairs of (6 • -2) -12 are (-3,4), (3,-4), (-6,2) (6,-2), (12,-1) and (-12,1). We can add each of these 6 pairs: 1, -1, -4, 4, 11 and -11. As there are 6 different factor pairs of -12, there are 6 different integer values of b.

Question 6 – Finding the different values of *c* from a and b

The n() means number of.

In order to answer this question, we have to consider the *addition* pairs instead of the factor pairs. In ax^2+bx+c, b is obtained from the factor pairs of ac. Then can the different values of c, be obtained from *a* and *b*? Let's use sets to illustrate this.

This is plainly used to simplify the idea. i.e. this is not the mathematical approach

1. As seen before … n(b) = n(factor pairs of ac)
2. To make things easier… n(b) = n(ac)
3. Can we rearrange this idea to… n(b) ÷ a = n(c)

What the 3rd line is showing is the number of different values of b that are divisible by a, equals to the number of different values of c.

What this illustration is trying to show, is that the number of pairs that add to b, when multiplied are divisible by c, is the amount of different values of c.

Lets call the pairs that add to the coefficient b, *p*. To qualify for a value of c, *p* must be multiplied together and then divisible by the value of a. If this results in an integer, this qualifies as 1 value of c. The amount of times this happens (same value of b, different addition pair) is the amount of different values of c.