Hollow Squares:

A Symmetric 960?

b (1st) Find total number of ways.
(2nd) How many are symmetrical?

(1) we are finding out a liberal difference of two squares.

$\therefore \quad x^2 - y^2 = 960 \quad \equiv (x+y)(x-y) = 960$

~~x~~ x, y : positive integers.

b Therefore, one can simply write out the factors.

960		
1	~~961~~	960
2	482	480
3	~~323~~	320
4	244	240
5	~~197~~	192
6	166	160
8	128	120
10	106	96
12	92	80
15	~~79~~	64
16	76	60
20	68	48
24	64	40
30	62	32
	the sum.	

b We know it can only work when the two factors sum to an even number

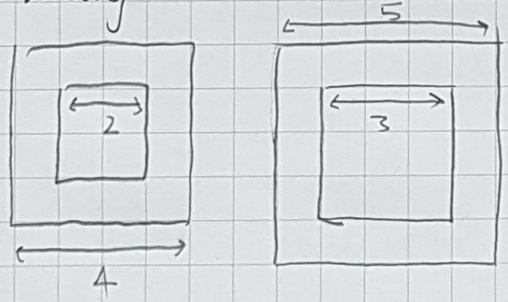
$\therefore \quad x+y+x-y = 2x$
$\equiv 2(x)$ $\therefore$ must be even.

b $\therefore$ can add the factors and eliminate the odd sums.

b As one can see, there are 10 possible ways to get 960 by taking away a smaller square from a larger square

b However, not all are symmetrical!

(2) Symmetry



We can see that symmetry is created when both numbers are either odd or even

b this is $\therefore$ odd $\pm$ odd = even
even $\pm$ even = even.

- Aim: filter out the the results for x and y (from $x^2 - y^2 = 960$) in which x and y are not both even or odd.

1) 2, 480
$2x = 482$
$x = 241$ odd
$y = 241 - 2 = 239$ odd.

2) 4, 240
$2x = 244$
$x = 122$ even
$y = 122 - 4 = 118$ even

3) 6, 160
$2x = 166$
$x = 83$ odd
$y = 83 - 6 = 77$ odd.

4) 8, 120
$2x = 128$
$x = 64$ even
$y = 64 - 8 = 56$ even.

5) 10, 96
$2x = 106$
$x = 53$ odd
$y = 53 - 10 = 43$ odd.

6) 12, 80
$2x = 92$
$x = 46$ even
$y = 46 - 12 = 34$ even.

7) 16, 60
$2x = 76$
$x = 38$ even
$y = 38 - 16 = 22$ even.

8) 20, 48
$2x = 68$
$x = 34$ even
$y = 34 - 20 = 14$ even.

9) 24, 40
$2x = 64$
$x = 32$ even
$y = 32 - 24 = 8$ even.

10) 30, 32
$2x = 62$
$x = 31$ odd
$y = 31 - 30 = 1$ odd.

Therefore, all 10 ways are symmetric hollow squares because you only get even pairs or odd pairs which produce a symmetric hollow square of 960.