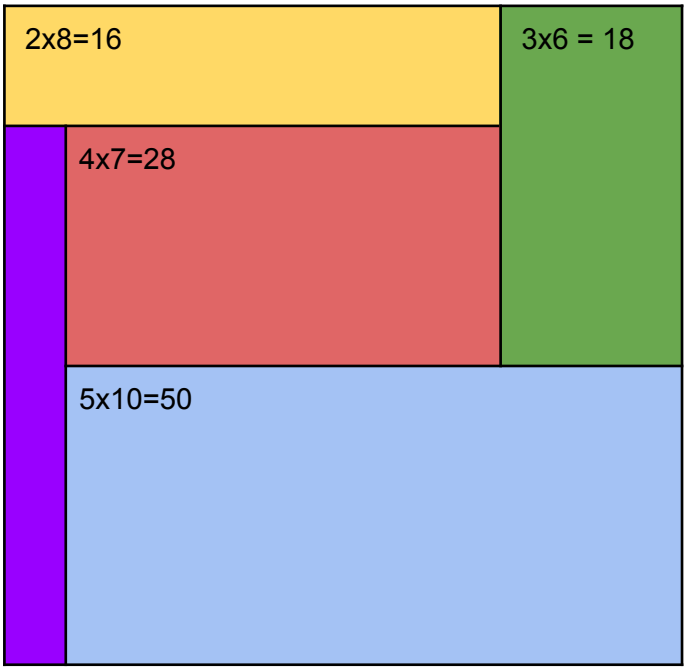
11x11 Square

Solution

The 1st and 3rd squares, as given in the problem, will not work as they have common sides of the same length.
Also, the 1st square has a side length of 11 which is not an option.

So I looked at the middle one and this is my solution:



(The purple rectangle is 1x9)
All of the rectangles add up to 121 or 11^2, the area of the square.

As you will see, my layout is similar to, but not exactly like the middle square in the problem. Let me show you my thinking:

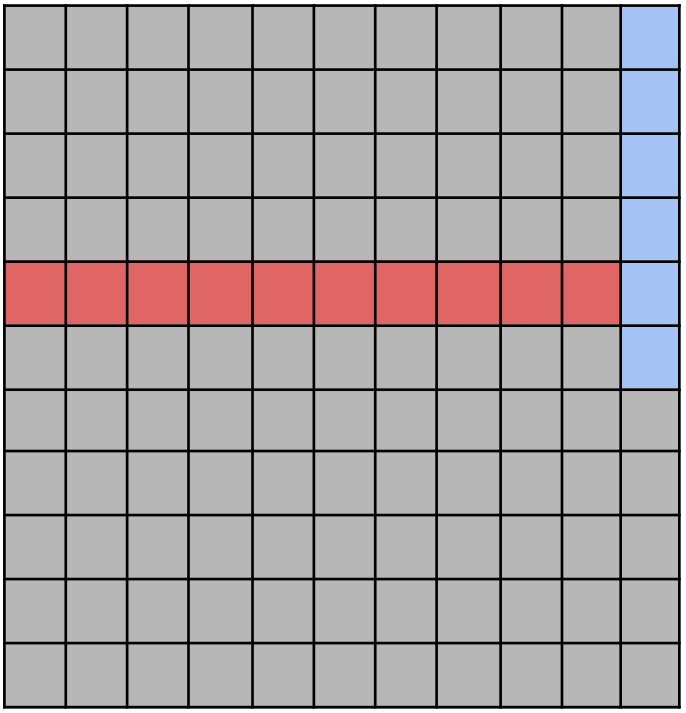
Step 1
I grouped numbers and created pairs. There are 5 pairs and they all add up to 11. These pairs are (1,10) (2,9) (3,8) (4,7) (5,6)

Why is this helpful?
Each side is made up of 2 measurements. The width of one rectangle and the length of a different rectangle. If it is a 11x11 square, we need to have the total length of each side of the square to be 11.

The key to this problem is that you cannot judge the measurements by simply looking at the square as given in the problem and trying to estimate from there.

Step 2
Find out the middle square (pink) So I started off by cancelling out the pairs that do not work in the middle.

The pair (1,10) doesn't work because the middle square would have to be set to one side and then there would be two rectangles with side length 1 (which is not allowed as per the problem).



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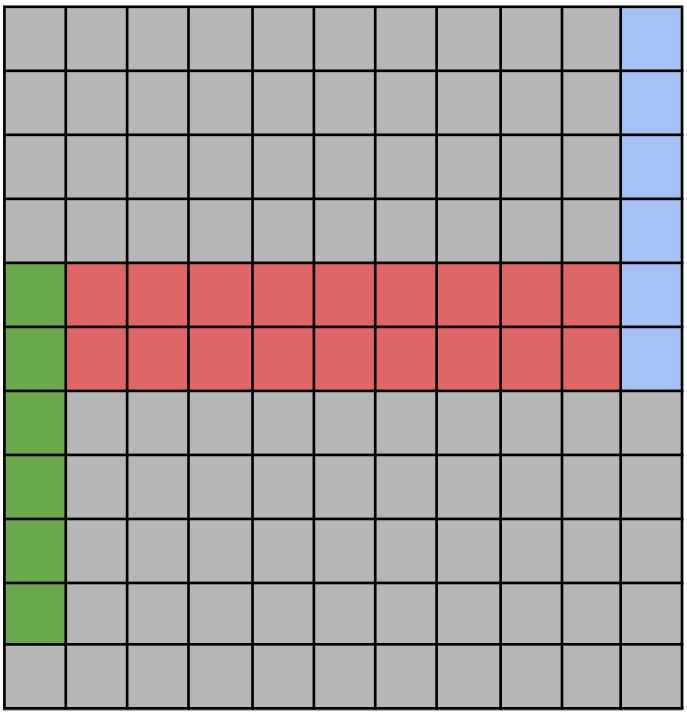
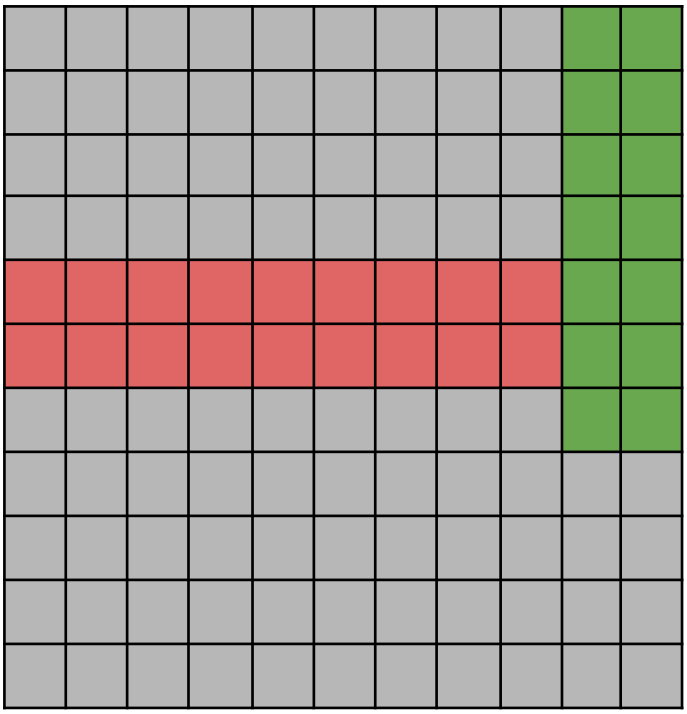
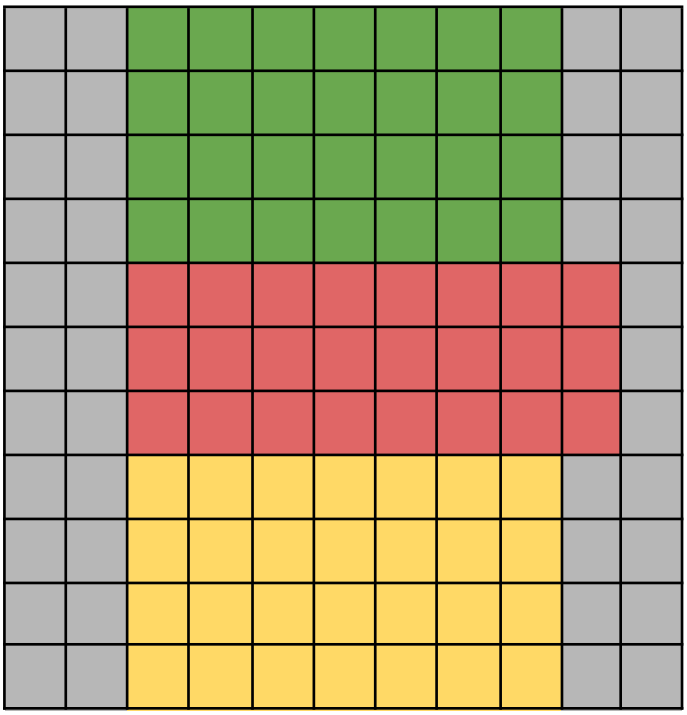
The pair (2,9) also doesn't work. If you put the middle rectangle in the centre, there will be 2 rectangles with the side length of 1.



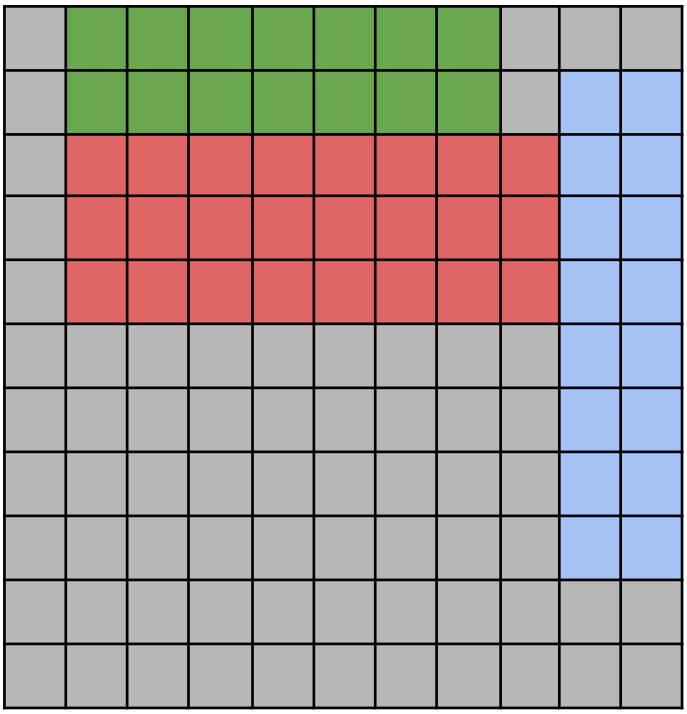
If you put the middle rectangle on the side there will be 2 rectangles with the side length of 2.



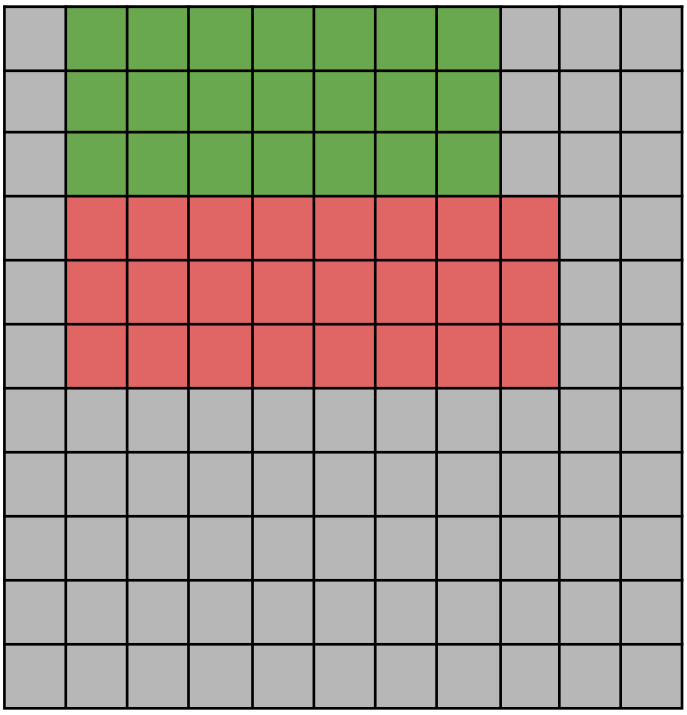
The next pair (3,8) also does not fit in the middle. When the middle rectangle is in the centre then we have two rectangles with lengths of 4.



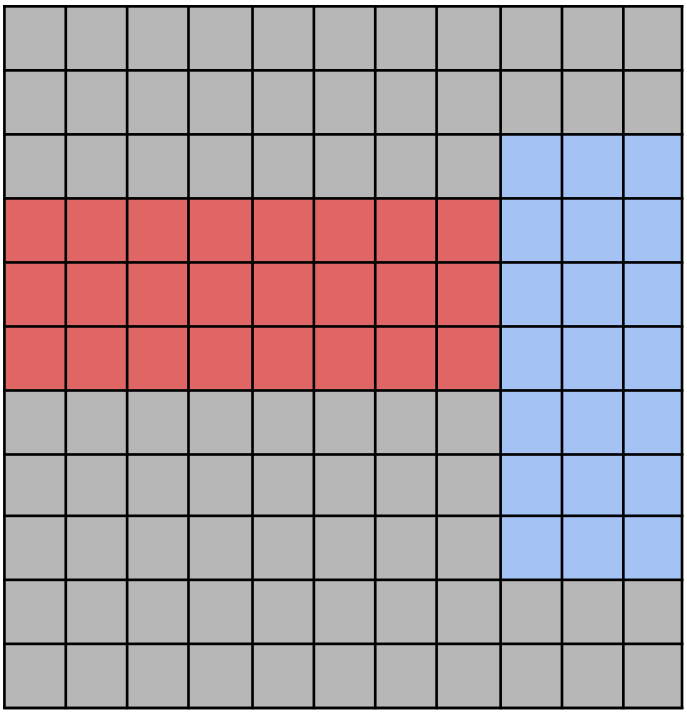
In this example the side length of 2 occurs twice and there are non-rectangular shapes being formed.



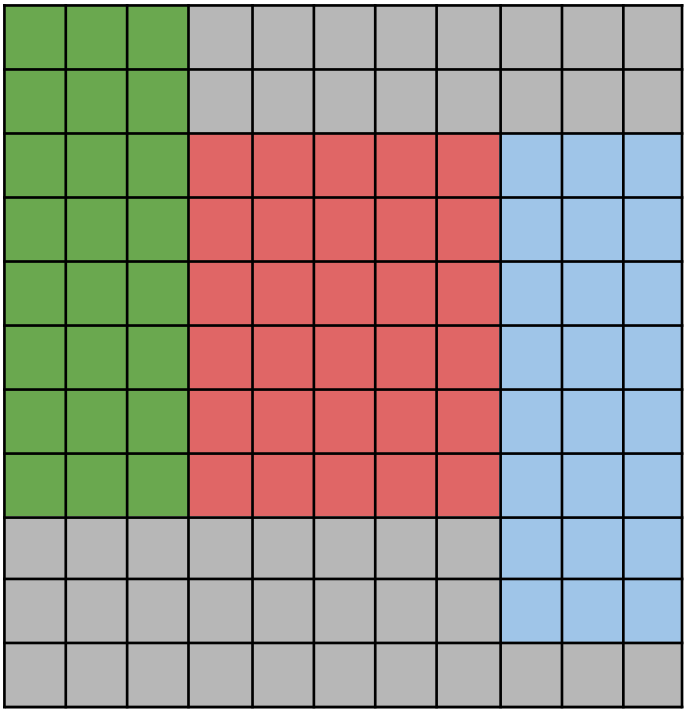
In this example there are 2 lengths of 3.



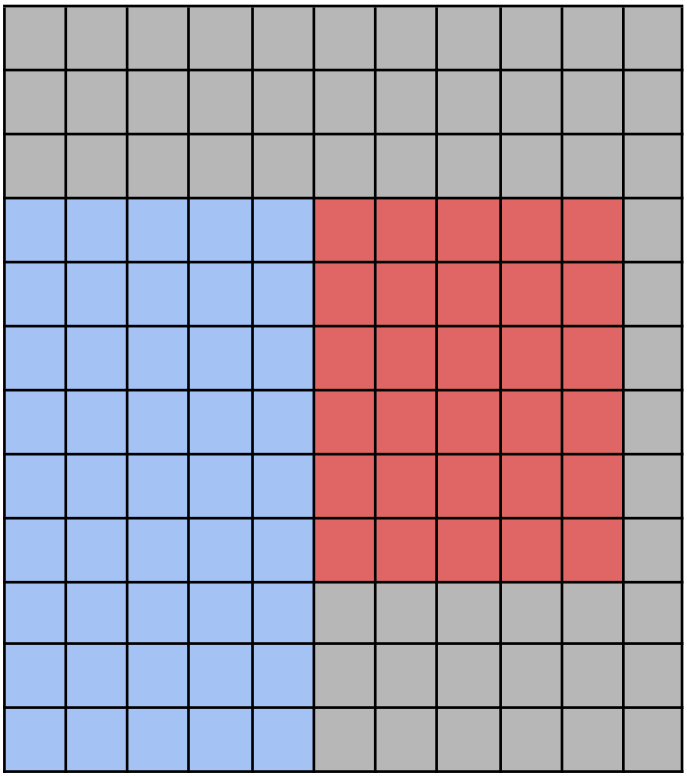
We cannot put the middle square on the side as there will be 2 side lengths of 3.



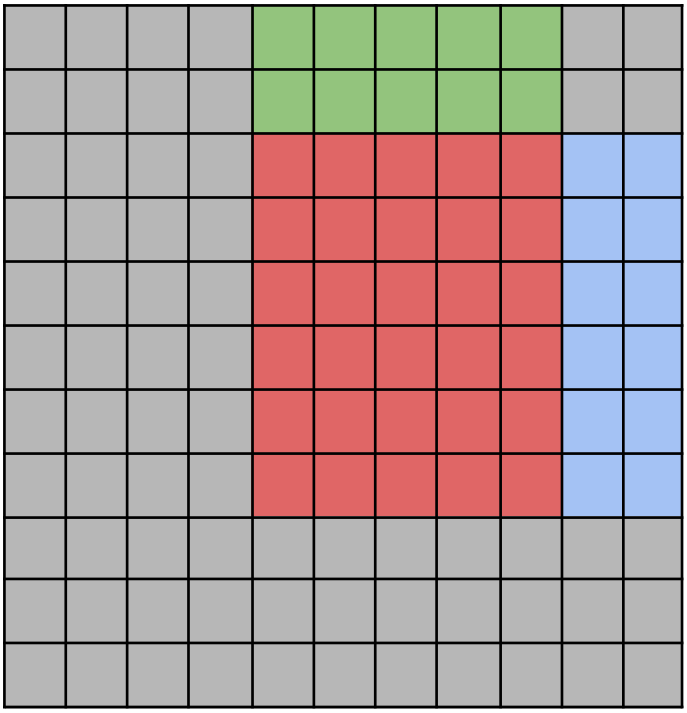
The next pair is 5,6 which also doesn't work. Placing the middle rectangle at the centre doesn't work as there will be 2 pairs of 3.



The middle rectangle placed here has the side length of 5 repeated twice.



The middle rectangle placed here has 2 repeated twice. If you move it down 1 unit then the bottom rectangle will have 2 instead of 3.



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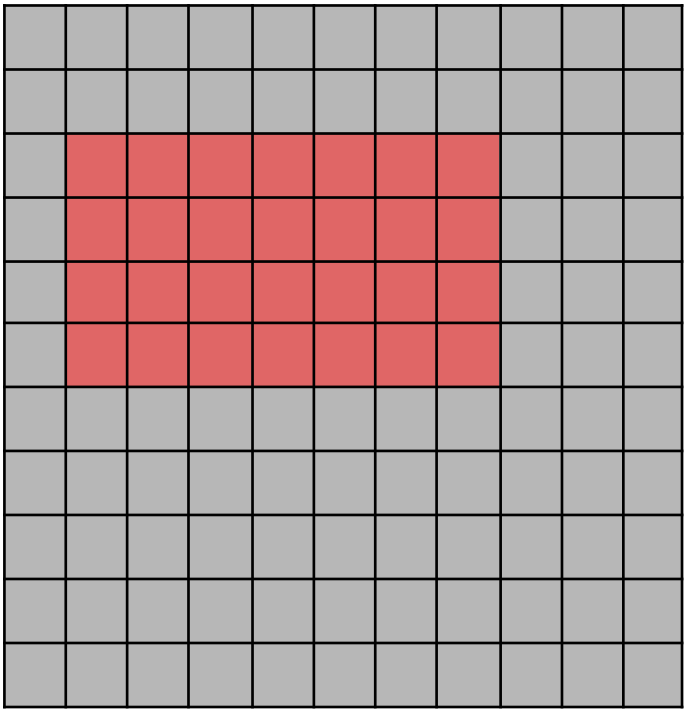
The final pair left is 4,7. And it does work!

There are many possibilities that don't work but there is one possibility that is not on the side and does work. The other possibilities that do work are rotations and translations. So I will show you the one that works according to the image.



This position can fit in all of the other sizes without any repetition of side length, as you can see below.

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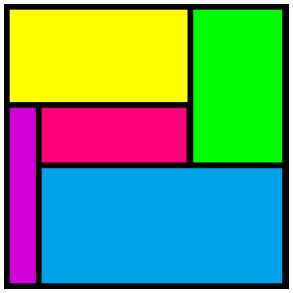
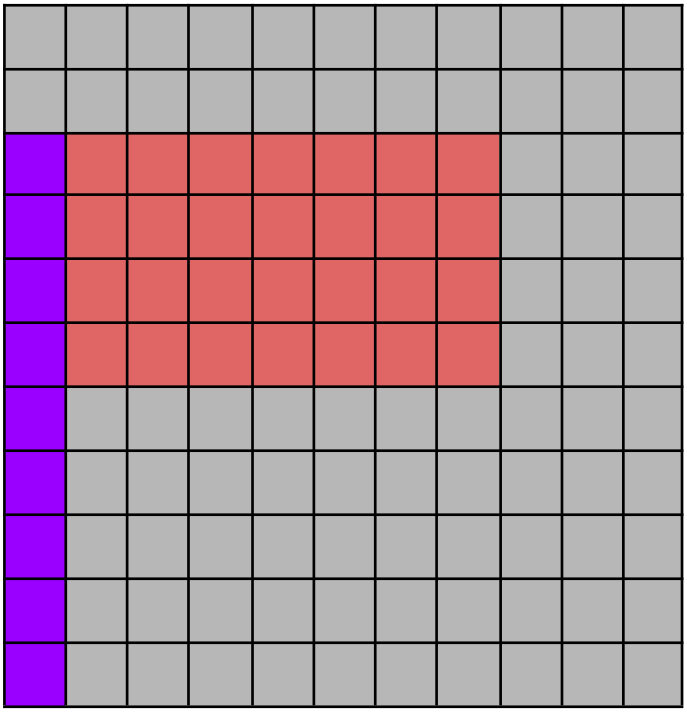
Step 3
Now we are going to solve the other sides



As we can see the purple rectangle starts at the top of the pink rectangle and ends at the bottom of the square.



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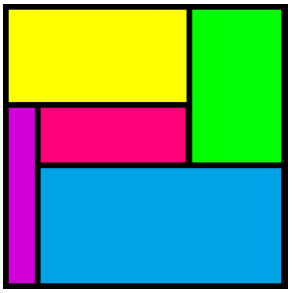
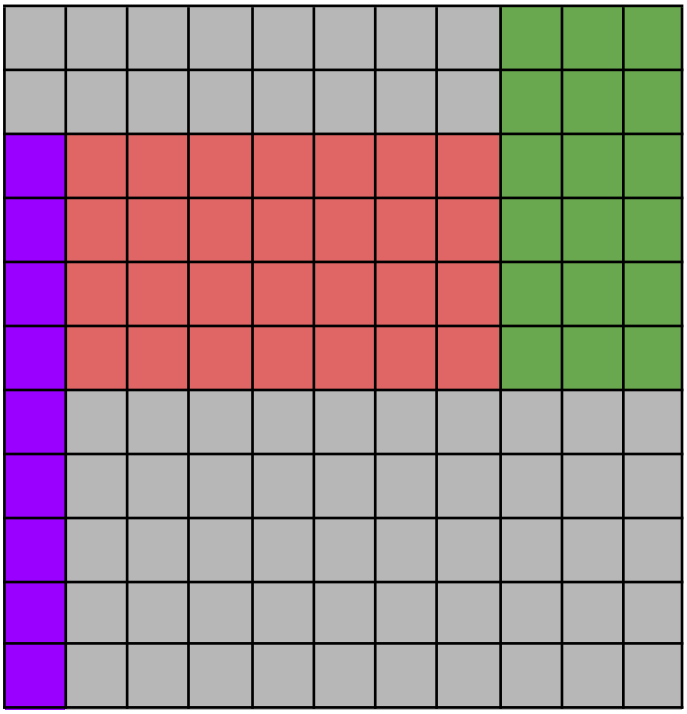
The green rectangle starts at the bottom of the pink rectangle and ends at the top of the square.





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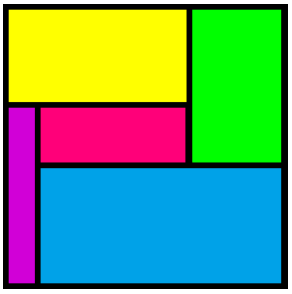
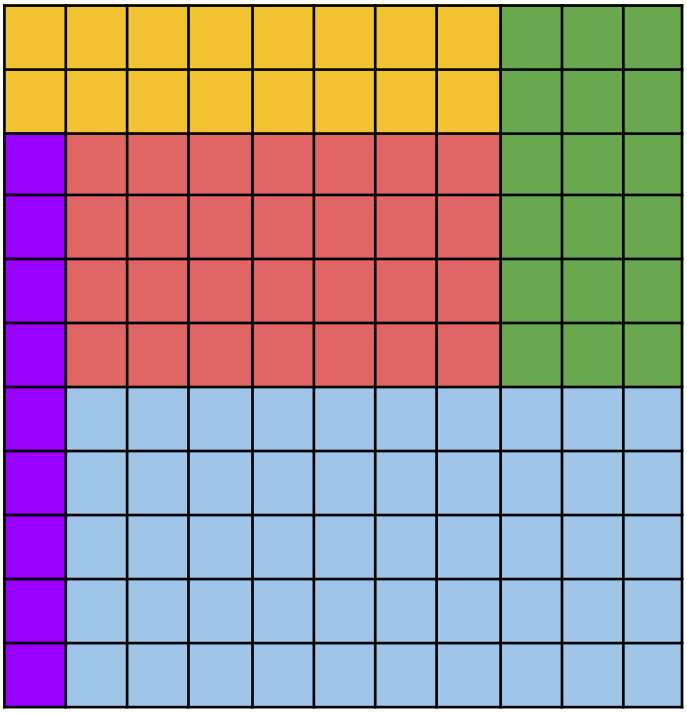
The top remaining rectangle would be yellow and the bottom rectangle would be blue like shown in the image.





How is this the correct solution?

The purple rectangle is 1x9
The yellow rectangle is 2x8
The green rectangle i s 3x6
The pink rectangle is 4x7
The blue rectangle is 5x10

There are no repeating measurements.
Like I showed, this is the only rectangle that works so in that case there is only one solution that is not a reflection or a rotation. As you can see, it looks a bit different from the image.