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Y10.

Difference of Two squares

Choose a number in the 3 times table. Take the numbers on either side of your chosen number and find the difference between their squares.

n	$3n$
0	0
1	3
2	6
3	9
4	12
5	15

$12+1 = 13 \qquad 13^2 = 169$

$12-1 = 11 \qquad 11^2 = 121$

~~121~~ $169 - 121 = 48$

Try it a few times. What do you notice?

$3+1 = 4 \qquad 4^2 = 16$

$3-1 = 2 \qquad 2^2 = 4$

$16 - 4 = 12$

$6+1 = 7 \qquad 7^2 = 49$

$6-1 = 5 \qquad 5^2 = 25$

$49 - 25 = 24$

Every time n increases, the difference of squares increases by 12 (for the 3 times table).

So the formula is:

$$(3n+1)^2 - (3n-1)^2 = 4 \times 3n = 12n.$$

Proven by:

$$(3n+1)^2 = (3n+1)(3n+1) = 9n^2 + 6n + 1$$

$$9n^2 + 6n + 1 - (9n^2 - 6n + 1)$$

$$= 12n.$$

This works for other times tables as well.

n	$5n$
0	0
1	5
2	10
3	15
4	20
5	25

$5+1=6 \qquad 6^2=36$

$5-1=4 \qquad 4^2=16 \qquad 36-16=20$

$10+1=11 \qquad 11^2=121$

$10-1=9 \qquad 9^2=81 \qquad 121-81=4$

For the 5 times table, every time n increases, the difference of squares increases by 20.

So the formula is:

$$(5n+1)^2-(5n-1)^2=4\times 5n=20n$$

Proven by:

$$\begin{aligned}(5n+1)^2 &= (5n+1)(5n+1)\\ &= 25n^2+10n+1\\ &\quad -(25n^2-10n+1)\\ &= 20n\end{aligned}$$

With this in mind, we notice a general patern in the times tables, by looking at the relationship between the times table and the formula's result.

n = any number $\quad t$ = times table.

n	tn
0	$t0$
1	$t1$
2	$t2$
3	$t3$
4	$t4$
5	$t5$

We can use the same format as for previo results.

So the formula is:

$$(tn+1)^2-(tn-1)^2=4\times tn$$

Proven by:

$$t^2n^2+2tn+1-(t^2n^2-2tn+1)$$

$$=4tn$$