

Mega Quadratic Equations

Consider each of the quadratics that make up the equation individually, and let their values be a and b . For the first equation:

$n^2-5n+5=a$, $n^2-11n+30=b$, and $a^b=1$. The possible solutions for $a^b=1$ are:

- $a=1$, b can be anything
- $a=-1$, b is an even number.
- $b=0$, $a \neq 0$.

These allow solving just one quadratic individually (and substituting the answers into the other quadratic to check that it works), which makes solving it much easier.

When $a=1$:

- $n^2-5n+5=1$
- $n^2-5n+4=0$
- $(n-1)(n-4)=0$
- $n-1=0$ or $n-4=0$
- $n=1$ or $n=4$

When $a=-1$:

- $n^2-5n+5=-1$
- $n^2-5n+6=0$
- $(n-2)(n-3)=0$
- $n-2=0$ or $n-3=0$
- $n=2$ or $n=3$
- When $n=2$, $n^2-11n+30=4-22+30=12$. 12 is an even number, so this is a solution.
- When $n=3$, $n^2-11n+30=9-33+30=6$. 6 is an even number, so this is a solution.

When $b=0$:

- $n^2-11n+30=0$
- $(n-6)(n-5)=0$
- $n-6=0$ or $n-5=0$
- $n=6$ or $n=5$
- When $n=5$, $n^2-5n+5=25-25+5=5$. It is not zero, so this is a solution.
- When $n=6$, $n^2-5n+5=36-30+5=11$. It is not zero, so this is a solution.

This gives $n=1$, $n=2$, $n=3$, $n=4$, $n=5$, and $n=6$ as the solutions for the first equation.

For the second equation, $n^2-7n+11=a$, $n^2-13n+42=b$, and $a^b=1$, as before. Take the same possibilities for a and b as with the first equation.

When $a=1$:

- $n^2-7n+11=1$
- $n^2-7n+10=0$
- $(n-2)(n-5)=0$
- $n-2=0$ or $n-5=0$
- $n=2$ or $n=5$

When $a=-1$:

- $n^2-7n+11=-1$
- $n^2-7n+12=0$
- $(n-3)(n-4)=0$
- $n-3=0$ or $n-4=0$
- $n=3$ or $n=4$
- When $n=3$, $n^2-13n+42=9-39+42=12$. 12 is an even number, so this is a solution.
- When $n=4$, $n^2-13n+42=16-52+42=6$. 6 is an even number, so this is a solution.

When $b=0$:

- $n^2-13n+42=0$
- $(n-6)(n-7)=0$
- $n-6=0$ or $n-7=0$
- $n=6$ or $n=7$
- When $n=6$, $n^2-7n+11=36-42+11=5$. It is not zero, so this is a solution.
- When $n=7$, $n^2-7n+11=49-49+11=11$. It is not zero, so this is a solution.

This gives $n=2$, $n=3$, $n=4$, $n=5$, $n=6$, and $n=7$ as solutions for the second equation.