Wipeout!

This problem is about investigating which number was crossed out from 1 to N with a specific mean left over.

Answer to a) At first there are six numbers, but after one being crossed out there are now only five numbers. If the mean of these is now 3.6, this means that their sum is:
3.6 x 5 = 18.
Now, what is the sum of the original numbers 1 2 3 4 5 6?
1+2+3+4+5+6 = 21.
21 – 18 = 3.
Therefore the number **3** was crossed out.
Answer to b) Let's try to use a similar method for this question. There are now only 6 numbers remaining, the mean of which is 4. Therefore the sum of the numbers left over is:
4 x 6 = 24.
The sum of the original numbers (using the ½ n(n+1) for the sum of the numbers from 1 to N) is 3.5(8) = 28. 28-24 = 4. Therefore the number crossed out must be **4**.
Answer to c) There are now only 14 numbers remaining, the mean of which is $7\frac{5}{7}$. Therefore the sum of the numbers left over is 14 x $7\frac{5}{7}$ = 108. The sum of numbers from 1 to 15 is 7.5(16) = 120.
120-108 = 12. Therefore the number crossed out was **12**.

Answer to d) There are now only ***N-1*** numbers remaining, the mean of which is 41/6, or 6 and 5/6. Therefore the sum of the numbers left over is $\frac{41\boldsymbol{N}-41}{6}$. The sum of the numbers from 1 to ***N*** is ½ ***N***(***N***+1). We know then, the number crossed out is (½ N^2 + ½ N) - $\frac{41\boldsymbol{N}-41}{6} = \frac{½\,\boldsymbol{N}(\boldsymbol{N}+1)}{1} - \frac{41(N-1)}{6} = \frac{3N(N+1)-41(N-1)}{6} = \frac{3N^2-38N+41}{6}$. Now, we know this expression equals the number crossed out. We need to find out which values this expression can take. What is the value when this expression equals zero? Then *N* = 11.4757… Via trial and error, we find that *N* = 13, and the number crossed out was 9.

Answer to e) There are now only ***N-1*** numbers remaining, the mean of which is $\frac{644}{25}$. Therefore the sum of the numbers left over is $\frac{644N-644}{25}$. The sum of the numbers from 1 to ***N*** is ½ ***N***(***N***+1). We know, then, the number crossed out is
$\frac{\frac{1}{2}N(N+1)}{1} - \frac{644(N-1)}{25} = \frac{\frac{25}{2}N(N+1)-644N+644}{25} = \frac{12.5N^2-631.5N+644}{25} = \frac{1}{2}N^2 - 25.26N + 25.76$. We know that the final expression equals the number crossed out. We need to find the value of *N* that makes this expression a whole number. When this expression = 0, *N* = 49.478744… Is there anything we can do to say what N can and can't be? We know that $\frac{1}{2}N^2 - 25.26N + 25.76 < N$. Therefore $\frac{1}{2}N^2 - 26.26N + 25.76 < 0$. This is turn means that N< The solution of this, which is slightly over 51. If we try N=51, it works, and the omitted number is 38.
Answer to extension) Let **m** be the mean of the remaining numbers. We know **m** is an integer. There are now only ***N-1*** numbers remaining, the mean of which is **m**. Therefore the sum of the numbers left over is **m(N-1)**. Note that this expression must also be an integer. The sum of the original numbers is ½ ***N***(***N***+1)**.** So the number (or numbers?) which must have been crossed out is
½ $\boldsymbol{N}^2$+ ½ ***N*** – **mN + m**. We've seen this in our previous examples. We can simplify this to
½ $\boldsymbol{N}^2$ +***N***(½ - m) + **m**. Either way, since we know ***N*** is an even number (we may assume it is positive), this expression, since it only involves ½ 's and no ¼ 's, is also whole. We know from previous

examples that $0 < \frac{1}{2}\boldsymbol{N}^2 + \frac{1}{2}\boldsymbol{N} - \mathbf{mN} + \mathbf{m} < \boldsymbol{N}$. Let's concentrate on the second inequality first. This means that $\frac{1}{2}\boldsymbol{N}^2 - \frac{1}{2}\boldsymbol{N} - \mathbf{mN} + \mathbf{m} < 0$, which is the same as $\boldsymbol{N}^2 - \boldsymbol{N} - 2\mathbf{mN} + 2\mathbf{m} < 0$. We can change this to $\boldsymbol{N}^2 - \boldsymbol{N}(1+2\mathbf{m}) + 2\mathbf{m} < 0$. What if this expression equalled zero? Then the solution to the equation would be $\boldsymbol{N} = \frac{(1+2m)\pm\sqrt{(1+2m)^2-8}}{2}$. So if we picked a value of ***N*** less than this solution value, the resulting answer would be less than zero. So all we need is that the value of ***N*** to be less than this solution value, which only depends on **m**. Ultimately, ***N*** and **m** also determine the wiped out number, so this is the relationship.