

## About Average

1. 3 numbers, mean = 3

Thus the sum of the three numbers  
 $3 \times 3 = 9$

Mode = 2

$$\boxed{\phantom{0}}, 2, 2, \boxed{\phantom{0}} \\ \boxed{\phantom{0}} = 9 - (2 + 2) = 5$$

Ans: 2, 2, 5

2. 3 numbers, mean = 7

Sum of the three numbers =  $7 \times 3 = 21$

Mode = 10

$$10, 10, \boxed{\phantom{0}} \\ \boxed{\phantom{0}} = 21 - (10 + 10) = 1$$

Ans: 1, 10, 10

3. 3 numbers, mean = 8

Sum of the three numbers =  $8 \times 3 = 24$

Median = 10

$$\boxed{a}, 10, \boxed{b}$$

$$a + b = 24 - 10 = 14 \quad \text{--- ①}$$

$$\text{Range } 8: b - a = 8 \quad \text{--- ②}$$

$$\text{①} + \text{②}: 2b = 14 + 8$$

$$b = 11$$

$$a = 14 - 11 = 3$$

Ans: 3, 10, 11

4. 4 numbers, mean = 7.5

Sum of the four numbers =  $7.5 \times 4 = 30$

mode = 6 IF 6 appears two times:

Case A :  $\boxed{a}$ , 6, 6,  $\boxed{b}$

OR

Case B : 6, 6,  $\boxed{a}$ ,  $\boxed{b}$

As sum = 30,  $a + b = 30 - (6 + 6) = 18$

Case A : Median = 6, not correct

Case B : Median = 7

$$6 + a = 7 \times 2 = 14$$

$$a + b = 18$$

$$a = 8, b = 10$$

mode = 6 IF 6 appears three times:

$\boxed{a}$ , 6, 6, 6

OR

6, 6, 6,  $\boxed{b}$

This case, median = 6 so this is not correct.

Ans : 6, 6, 8, 10

5. 4 numbers, mean = 6

Sum of the four numbers =  $6 \times 4 = 24$

median = 6.5

$$[a] + [b] + [c] + [d] = 24$$

$$\frac{b+c}{2} = 6.5$$

$$b+c = 13$$

$$a+d = 24 - 13 = 11 \text{ --- } \textcircled{1}$$

As range = 11,  $d - a = 11 \text{ --- } \textcircled{2}$        $\textcircled{1} + \textcircled{2} : 2d = 22 \quad d = 11 \quad a = 0$   
 $0, \square, \square, 11$       ~~No~~

Since all numbers must be possible positive integers, there's no possible answer.

6. 5 numbers, mean = 4

Sum of the five numbers =  $4 \times 5 = 20$

IF 3 appears three times:

3, 3,  $[a]$ ,  $[b]$ ,  $[c]$

← Not correct because  $a+b=2$  and so it is not less than 3.

$\textcircled{1}$  ---  $[a]$ , 3, 3,  $[b]$ ,  $[c]$

$\textcircled{2}$  ---  $[a]$ ,  $[b]$ , 3, 3,  $[c]$

$\square, \square, \square, 3, 3$  ← Not correct because range = 9

$$\textcircled{1} \quad a+b+c = 14$$

$$c - a = 9$$

$$b + 2c = 23$$

$$a = c - 9$$

| b | c  | b+2c | a  |
|---|----|------|----|
| 3 | 10 | 23   | 1  |
| 5 | 9  | 23   | 0  |
| 7 | 8  | 23   | -1 |

← can't be negative.

$$\textcircled{2} \quad b \quad c \quad b+2c \quad a$$

|   |    |    |   |
|---|----|----|---|
| 1 | 11 | 23 | 2 |
| 3 | 10 | 23 | 1 |

← Not correct because a is bigger than b.

Ans: 1, 3, 3, 3, 10

7. 5 numbers, mean = 4

Sum of the five numbers =  $4 \times 5 = 20$

IF 2 appears two times:

① ... 2, 2,  $\boxed{a}$ ,  $\boxed{b}$ , 8

$\boxed{1}$ , 2, 2,  $\boxed{1}$ ,  $\boxed{7}$  ← Not correct because sum must be 20.

$\boxed{1}$ ,  $\boxed{7}$ , 2, 2,  $\boxed{1}$  ← Not correct because 1 should NOT appear two times.

$\boxed{1}$ ,  $\boxed{1}$ ,  $\boxed{1}$ , 2, 2 ← Not correct because range must be 6.

①  $a + b = 8$       2, 2,  $\boxed{a}$ ,  $\boxed{b}$ , 8

3

5

2

6

4

4

← 4 should not appear two times as mode = 2.

Ans: 2, 2, 3, 5, 8 or

2, 2, 2, 6, 8

8. 5 numbers, mean = 7

Sum of the five numbers =  $7 \times 5 = 35$

IF 7 appears two times:

7, 7,  $\boxed{a}$ ,  $\boxed{b}$ ,  $\boxed{17}$  ← Not correct because  $a + b = 4$  and it can't be less than 7.

① ...  $\boxed{a}$ , 7, 7,  $\boxed{b}$ ,  $\boxed{c}$

① ...  $\boxed{a}$ ,  $\boxed{b}$ , 7, 7,  $\boxed{c}$

$\boxed{1}$ ,  $\boxed{1}$ ,  $\boxed{1}$ , 7, 7 ← Not correct because range must be 10.

①  $a + b + c = 21$

$c - a = 10$        $c = a + 10$

$a + b + a + 10 = 21$

$2a + b = 21 - 10$

$2a + b = 11$

| a | b | 2a+b | c  |
|---|---|------|----|
| 1 | 9 | 11   | 11 |
| 2 | 7 | 11   | 12 |
| 3 | 5 | 11   | 13 |

Ans: 1, 7, 7, 9, 11

2, 7, 7, 7, 12

3, 5, 7, 7, 13