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NRICH

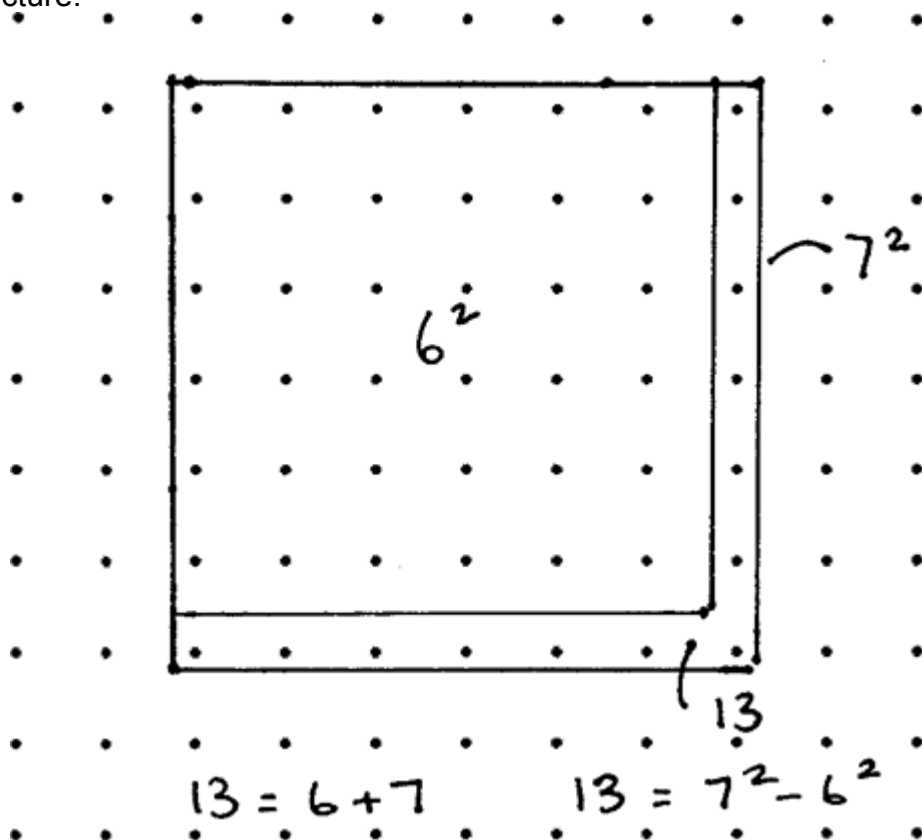
Odd Differences

Age 14 to 16 Challenge Level:

Tom started us off with the following idea. Thanks Tom.

In the square of dots you can imagine the "L" of any odd number and if you take the square for the odd number below it from the square of the odd number you get the number you stated with. To find the two squares you have to find half the odd number and round it up and down. So half of 13 is 6.5 so the two squares have sides 7 and 6.

Here is a picture:



So, using the diagram you know that

$$105 = \frac{105 - 1}{2} + \frac{105 + 1}{2}$$

$$105 = 52 + 53$$

$$105 = 53^2 - 52^2$$

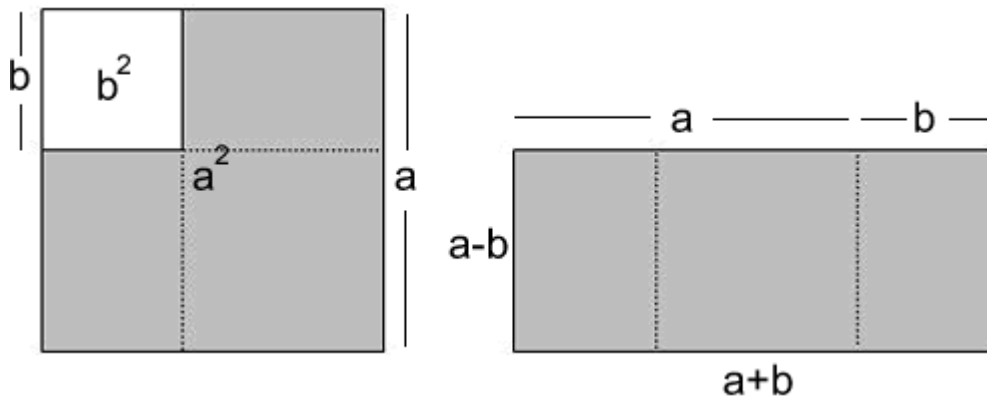
You can always find one way using this method. So

$$1155 = \frac{1155 - 1}{2} + \frac{1155 + 1}{2}$$

$$1155 = 577 + 578$$

$$105 = 578^2 - 577^2$$

Alex, from the Grammar School at Leeds extends the algebra building on the second visualisation:



The task is to find eight pairs of numbers (a, b) , satisfying the following equation:

$$a^2 - b^2 = 1155$$

The LHS can be factorized to $(a + b)(a - b)$ so now we can begin to find eight factor pairs of 1155:

Using prime factorization,

$$1155 = 5 \times 231 = 5 \times 7 \times 33 = 5 \times 7 \times 3 \times 11$$

Now we use different selections of these prime factors to build the list of eight factor pairs:

1155

$$\begin{aligned} 1155 &= 5 \times 231 \\ &= 7 \times 165 \\ &= 3 \times 385 \\ &= 35 \times 33 \\ &= 21 \times 55 \\ &= 15 \times 77 \\ &= 11 \times 105 \\ &= 1 \times 1155 \end{aligned}$$

1155

Then it is simply a matter of substituting each pair into the following procedure:

$$1155 = (a + b)(a - b) \Rightarrow \text{Large factor} \times \text{small factor}$$

$$a + b = \text{large factor}$$

$$a - b = \text{small factor}$$

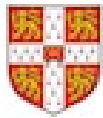
$$2b = \text{large factor} - \text{small factor}$$

$$b = (\text{large} - \text{small})/2$$

$$a = b + \text{small factor}$$

Applying this to each pair:

$$\begin{aligned}1155 &= 46^2 - 31^2 \\ &= 58^2 - 47^2 \\ &= 578^2 - 577^2 \\ &= 118^2 - 113^2 \\ &= 86^2 - 79^2 \\ &= 194^2 - 191^2 \\ &= 34^2 - 1^2 \\ &= 38^2 - 17^2\end{aligned}$$



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