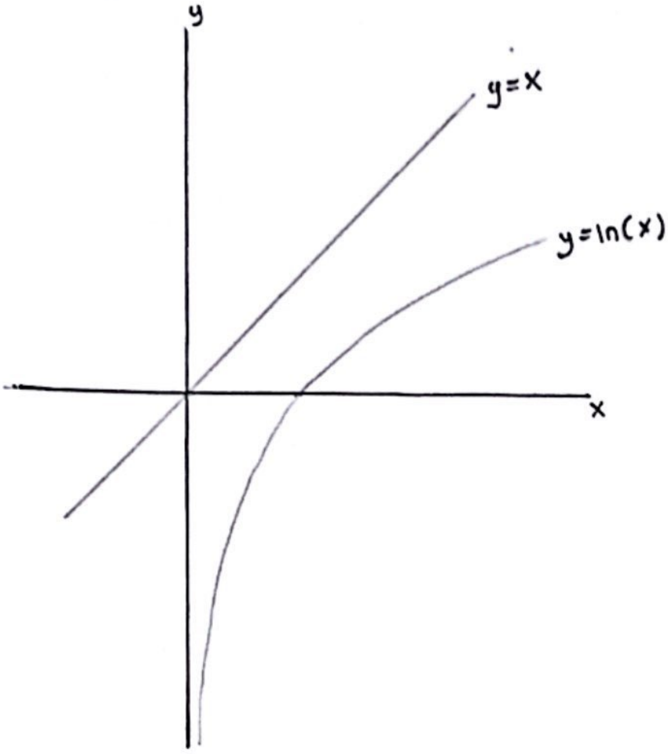
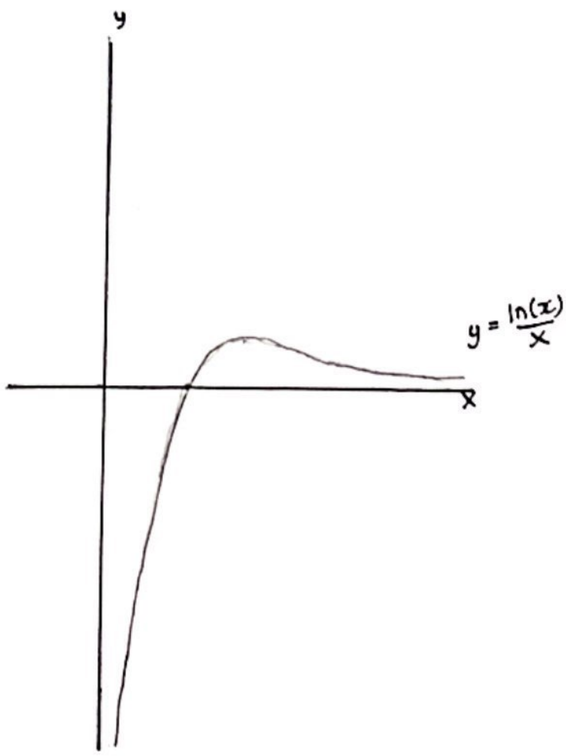
Deriving $y = \frac{\ln(x)}{x}$ using $\ln(x)$ and x



Since $y = x$ is a linear equation and $\ln(x)$ is a logarithmic function of base e, the graph of $\ln(x) \div x$ will reach a peak but instead of increasing to infinity it will aproach the x-axis.



$n^m = m^n$

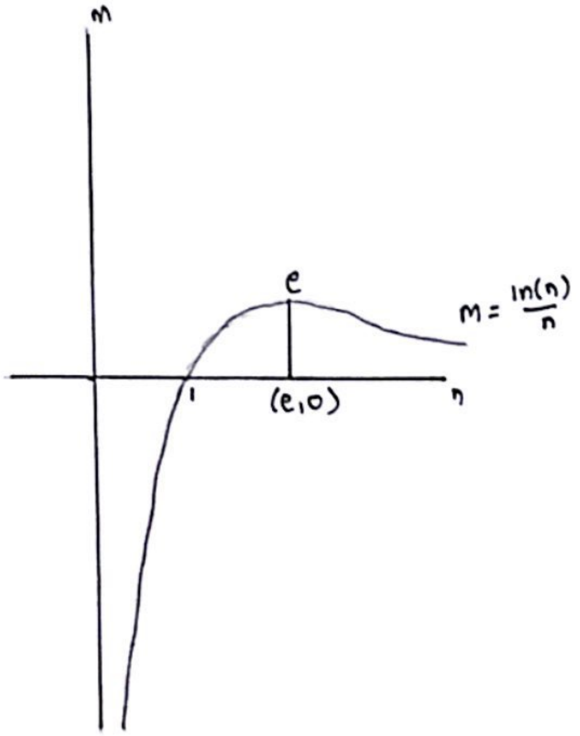
$\ln n^m = \ln m^n$

$m \ln(n) = \ln(m^n)$

$m \ln(n) = n \ln(m)$

$$\frac{\ln(n)}{n} = \frac{\ln(m)}{m}$$

$$\left(\frac{a}{b}\right)' = \frac{(a' \cdot b) - (b' \cdot a)}{a^2}$$



By using Calculus we can find the maximum point of a graph by

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dn}\left(\frac{\ln(n)}{n}\right) = \frac{\left(\frac{d}{dx}\ln(n) \cdot n - \frac{d}{dx} n \cdot \ln(n)\right)}{n^2}$$

$$= \frac{\frac{1}{n} \cdot n - 1 \cdot \ln(n)}{n^2}$$

$$= \frac{1 - \ln(n)}{n^2}$$

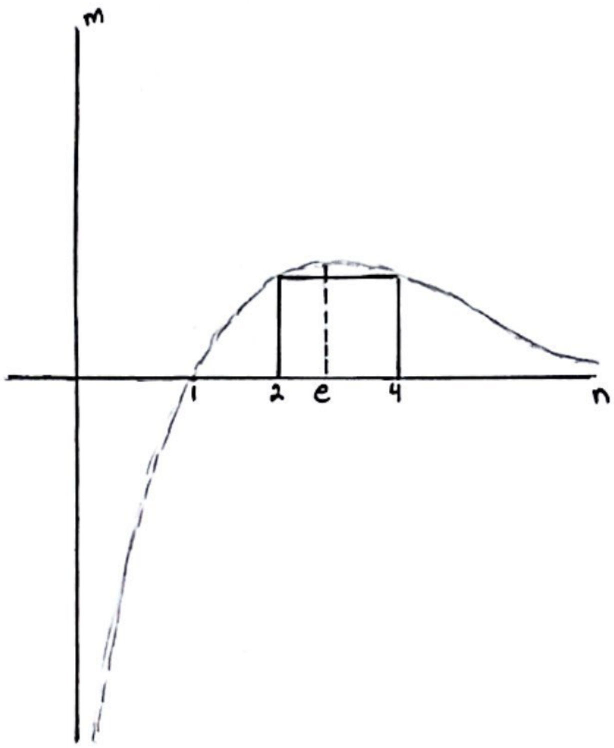
$\frac{dy}{dn} = 0$ will yield the max point

$1 - \ln(n) = 0$

$1 = \ln(n)$

$n = e$

Using the idea of the graph on page 2, we can see the peak occurs when $n=e$, and since we are looking for solutions in $n \in \mathbb{Z}^+$, the solutions will lie on either side of the maximum. therefore $n>e$ and $m<e$ and vice versa. Therefore to find these it will be enough to consider $n=0,1,2$.



n can be ether 1 or 2, however by graphing we see that if $n=1$, then the other solution wont lie on the curve, but instead somewhere on the x-axis, however the x-axis is a asymptote so a solution can never occur when $n=1$.

This means that n must equal to 2 for the integer solution, and by drawing a horizontal line to the other value, we obtain 4. (both occur at 0.3465735).

Therefore we can say that the only values (m,n) or (n,m) for which their is a integer solution fitting $n^m = m^n$ for $m \neq n$ are

$(2,4)$ and $(4,2)$

we can use the log laws to prove that

$$\frac{\ln(2)}{2} = \frac{\ln(4)}{4} \quad , \quad \underline{\underline{2^4 = 4^2}}$$

$$\frac{\ln(2)}{2} = \frac{\ln(4)}{4} \quad \Rightarrow \quad \log a^b = b \log a$$

$$\frac{\ln(2)}{2} = \frac{\ln(2^2)}{4}$$

$$\frac{\ln(2)}{2} = \frac{2\ln(2)}{4}$$

$$\frac{\ln(2)}{2} = \frac{\ln(2)}{2}$$

For this reason the graph maps the vaves of 2 and 4 at 0.3465735903 or both at $\frac{\ln(2)}{2} = y$.

We can provide further proof that is the case by graphing $y = \frac{\ln(x)}{x}$ and $y = \frac{\ln(2)}{2}$, to show intersections at $(2, \frac{\ln(2)}{2})$ and $(4, \frac{\ln(2)}{2})$.